Strong evidence for an accelerating universe

Nicola Vittorio
in collaboration with
B.S. Haridasu, V. Luković and R.D’Agostino

May 9, 2017
Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonscbe Differentialgleichung

$$\Delta \phi = 4 \pi K \phi$$  \hspace{1cm} (1)

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonscbe Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential \( \phi \) einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen hinzutreten für das räumlich Unendliche, falls man die Welt wirklich als räumlich unendlich ausgedehnt anzusehen hat.

Bei der Behandlung des Planetenproblems habe ich diese Grenzbedingungen in Gestalt folgender Annahme gewählt: Es ist möglich, ein Bezugssystem zu wählen, das räumlich Gravitationspotential...
A. Friedmann

Friedmann models: 1922, 1924

Strong evidence for an accelerating universe
UN UNIVERSE HOMOGENE DE MASSE CONSTANTE ET DE RAYON CROISSANT,
RENDAINT COMPTE
DE LA VITESSE RADIALE DES NEBULEUSES EXTRA-GALACTIQUES

Note de M. l’Abbé G. Lemaître

1. GÉNÉRALITÉS.
La théorie de la relativité fait prévoir l’existence d’un univers homogène où non seulement la répartition de la matière est uniforme, mais où toutes les positions de l’espace sont équivalentes, il n’y a pas de centre de gravité. Le rayon R de l’espace est constant, l’espace est elliptique de courbure positive uniforme 1/R², les droites issues d’un même point repassent à leur point de départ après un parcours égal à πR, le volume total de l’espace est fini et égal à π²R³, les droites sont des lignes fermées parcourant tout l’espace sans rencontrer de frontière (1).

Deux solutions ont été proposées. Celle de DE SITTER ignore la présence de la matière et suppose sa densité nulle. Elle conduit à certaines difficultés d’interprétation sur lesquelles nous aurons l’occasion de revenir, mais son grand intérêt est d’expliquer le fait que les nébuleuses extra-galactiques semblent nous fuir avec une énorme vitesse, comme une simple conséquence des propriétés du champ de gravitation, sans supposer que nous nous trouvons en un point de l’univers doué de propriétés spéciales.

L’autre solution est celle d’EINSTEIN. Elle tient compte du fait évident que la densité de la matière n’est pas nulle et elle conduit à une relation entre cette densité et le rayon de l’univers. Cette relation a fait prévoir l’existence de l’univers d’un rayon croissant, ce qui est aujourd’hui confirmé.
EFFECT OF INHOMOGENEITY ON COSMOLOGICAL MODELS

By Richard C. Tolman

Norman Bridge Laboratory of Physics, California Institute of Technology

Communicated February 12, 1934

1. Introduction.—In the application of relativistic mechanics and relativistic thermodynamics to cosmology, it has been usual to consider homogeneous models of the universe, filled with an idealized fluid, which at any given time has the same properties throughout the whole of its spatial extent. This procedure has a certain heuristic justification on account of the greater mathematical simplicity of homogeneous as compared with non-homogeneous models, and has a measure of observational justification on account of the approximate uniformity in the large scale distribution of extra-galactic nebulae, which is found out to the some $10^8$ light-years which the Mount Wilson 100-inch telescope has been able to penetrate. Nevertheless, it is evident that some preponderating tendency for inhomogeneities to disappear with time would have to be demonstrated, before such models could be used with confidence to obtain extrapolated conclusions as to the behavior of the universe in very distant regions or over exceedingly long periods of time.

It is the object of the present note to contribute to our knowledge of the effects of inhomogeneity on the theoretical behavior of cosmological models. For the immediate purposes of this investigation we shall confine our attention to very simple models composed of dust particles (nebulae) which exert negligible pressure and which are distributed non-uniformly...
SPHERICALLY SYMMETRICAL MODELS IN GENERAL RELATIVITY

H. Bondi

(Received 1947 August 5)

Summary

The field equations of general relativity are applied to pressure-free spherically symmetrical systems of particles. The equations of motion are determined without the use of approximations and are compared with the Newtonian equations. The total energy is found to be an important parameter, determining the geometry of 3-space and the ratio of effective gravitating to invariant mass. The Doppler shift is discussed and is found to contain both the velocity shift and the Einstein shift combined in a rather complex expression.

1. Introduction.—The field equations of the general theory of relativity are very complex. The only non-static solutions which have so far been obtained are either approximations or are of the cosmological type. Since approximate solutions apply only in the cases where the field is almost Newtonian, their use in pointing out intrinsic consequences of the theory is somewhat restricted. Similarly cosmological solutions suffer from the disadvantage that the spatial part of space-time is supposed to be homogeneous and isotropic. Therefore it is often difficult, owing to the lack of independent variables, to disentangle the causes of various effects.

The main purpose of the present paper is to derive the equation of motion and to describe various properties of pressure-free spherically symmetrical systems. A rigorous solution of the field equations has been obtained and it is hoped that the model presented can be of use in illustrating and clarifying various points of interest in the theory.

The results are considered to be an extension of the work of McCrea, by H. Bondi.
attributed not only to the action of the rennet, but also, as our papers about to appear show, to the elaboration of a peptic-like enzyme by certain lactic acid streptococci. Even within the first twenty-four hours of ripening, the amount of subpeptone appearing suggests that associated with the peptolic action is a tryptic-like action—a conjecture that again in the light of our cultural studies on certain other lactic acid streptococci is not without merit.

Subject to qualification as further data on the nature of specific enzymes may appear, the results of our study show that after the first few hours of ripening, the proteolytic breakdown in the ripening of Kingston cheese is of an associative peptic-tryptic-like nature.

This study of nitrogen distribution is one of a series on cheese-ripening which is provided for by a research fund established jointly by the Empire Marketing Board and the University of British Columbia. A detailed account of the experiments will appear shortly in the Journal of Dairy Research, Cambridge.

G. Lemaître

University of British Columbia, Vancouver, Canada, Mar. 27.

Insect Remains in the Gut of a Cobra, Naja tripudians.

The accompanying photograph (Fig. 1) shows the remains of insects belonging to three orders, namely, Elateridae (Heteroptera-Pentatomidae), Coleoptera, and Hymenoptera (Formicoidea), found in the gut of a cobra, Naja tripudians, brought to us in November 1926. The cobra, which was the black variety with no markings on the back of the hood but with white

come to our notice being that of "a small reddish beetle." Found in the gut of Tropidoclonius annulatus (Wall and Evans, Journal Bombay Nat. Hist. Soc., vol. 13) and of a locust (the species not stated) eaten on one occasion by Echis carinata (Wall, id., vol. 18).

The parts of the pentatomid comprise a right hemelytron, pronotum, sternum, scutum, and pygophor. The ants are ponerine and are capable of inflicting a nasty sting.

We are indebted to Dr. T. A. Buckley, Forest Department, S.S., and F.M.S., for assistance in preparing the photograph.

N. C. E. Miller.

H. T. Paeden.

Department of Agriculture.

Suck's Settlements and

Federated Malay States.

The Beginning of the World from the Point of View of Quantum Theory.

Sir Arthur Eddington states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical principles from the point of view of quantum theory may be stated as follows: (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed in a few or even in a unique quantum.

Now, in atomic processes, the notions of space and time are no more than statistical notions, they fade out when applied to individual phenomena involving a small number of quanta. If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

It may be difficult to follow up the idea in detail as we are not yet able to count the quantum packets in
Nous ne nous proposons pas dans ce travail de discuter les hypothèses sur lesquelles se fonde la théorie de l'expansion de l'Univers, ni la valeur des confirmations astronomiques qui l'étayent. Une telle discussion nous paraît actuellement prématurée et ne pourrait certes pas arriver à des conclusions définitives dans l'état actuel de la théorie et des observations.

La théorie peut être développée de deux façons : par l'étude de solutions exactes des équations de la gravitation, fournissant des modèles simplifiés, ou par le développement approché de la solution de problèmes plus complexes. Il nous paraît utile de ne pas mêler ces deux méthodes, et dans ce travail nous ne nous occuperons que de solutions mathématiquement...
Friedmann - Lemaître - Robertson - Walker models

- FLRW metric
  \[ ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \]

- Field equations
  \[
  H^2(z) = H_0^2 \left[ \sum_i \Omega_i (1 + z)^{3(1+w_i)} + \Omega_k (1 + z)^2 \right]
  \]
  \[
  \dot{H} + H^2 = -\frac{H_0^2}{2} \sum_i \Omega_i (1 + 3w_i)(1 + z)^{3(1+w_i)}
  \]

- Cosmological parameters
  \[
  \Omega_k \equiv -kc^2/[H_0^2 R(t_0)^2]
  \]
  \[
  \Omega_i \equiv 8\pi G\rho_i/(3H_0^2)
  \]
  \[
  p = w_i(\rho_i c^2)
  \]
Cosmological parameters: Planck results (Ade et al. 2016)

- $\Omega_b = 0.04949 \pm 0.0005$
- $\Omega_m = 0.315 \pm 0.013$
- $\Omega_\Lambda = 0.685 \pm 0.013$
- $\Omega_k = 0$
- $H_0 = (67.31 \pm 0.96) \text{km s}^{-1} \text{Mpc}^{-1}$

Low-redshift accelerated expansion: $\Omega_\Lambda > \Omega_m/2$
Accelerated expansion

- First evidence from the SN Ia
  - Riess et al. 98
  - Perlmutter et al. 99

Strong evidence for an accelerating universe
Cosmic acceleration

- Further confirmed by
  - the most recent supernova data
    - Betoule et al. 14
  - Observational Hubble parameter-OHD
    - Jimenez et al. 02
  - Baryon Acoustic Oscillations-BAO
    - Eisenstein et al. 05

- Consistency with the Planck measurements
  - Ade et al. 16

- A flat $\Lambda$CDM has been "established" as the concordance model

Strong evidence for an accelerating universe
The JLA dataset (Betoule et al. 14)

- Consists of 740 SN
- Corrections for galactic extinction
- From the observed to the SN rest-frame B band.
- Processed using SALT light curve and spectral fitters
  - (Guy et al. 07)
- For each SN:
  - $z$
  - $m_B$: B-band apparent magnitude at the maximum
  - $s$: stretch correction
  - $c$: colour correction

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Strong evidence for an accelerating universe
Magnitude-redshift relation

- Luminosity distance: \( d_L = d_H D_L \)
- Hubble radius: \( d_H \equiv c/H_0 \simeq 3000 \, h^{-1} \, \text{Mpc} \)
- “Hubble constant-free” luminosity distance: \( D_L \).
  - It depends on the other cosmological parameter

\[
\mathcal{M} \equiv M_B^{\text{corr}} + 5 \log_{10} d_H
\]

Then, the magnitude-redshift relation becomes

\[
m_B = \mathcal{M} - \alpha s + \beta c + 5 \log_{10} D_L(\Omega_m, w) + 25
\]

This equation is used to fit at once
- the three nuisance parameters \( (\alpha, \beta, \mathcal{M}) \)
- the cosmological parameters

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Strong evidence for an accelerating universe
Nielsen et al. 15 proposed to consider a Normal distribution for

- the SN absolute magnitude $\mathcal{N}(M_0, \sigma_{M_0}) \iff \mathcal{N}(M_0, \sigma_{M_0})$
- the colour $\mathcal{N}(c_0, \sigma_{c_0})$ and stretch $\mathcal{N}(s_0, \sigma_{s_0})$ corrections

There are 8 nuisance parameters

$$\alpha, \beta, M_0, \sigma_{M_0}, c_0, \sigma_{c_0}, s_0, \sigma_{s_0}$$

plus the cosmological parameters.

The joint probability for the intrinsic SN parameters is

$$p(Y) = |2\pi \Sigma_l|^{-1/2} \exp[-(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^T / 2]$$

- $Y = (M_1, s_1, c_1, \ldots, M_N, s_N, c_N)$ is a $3N$-vector with the parameters of each of the $N$ supernovae;
- $Y_0 = (M_0, s_0, c_0, \ldots, M_0, s_0, c_0)$ is a $3N$-vector with the central values of the parameters’ distributions;
- $\Sigma_l = \text{diag}(\sigma_{M_0}^2, \sigma_{s_0}^2, \sigma_{c_0}^2, \ldots, \sigma_{M_0}^2, \sigma_{s_0}^2, \sigma_{c_0}^2)$ is the $3N \times 3N$ covariance matrix.
Comparison with the theoretical models

- $\hat{Z} = (\hat{m}_{B1} - \mu^{th}(z_1), \hat{s}_1, \hat{c}_1, \ldots, \hat{m}_{BN} - \mu^{th}(z_N), \hat{s}_N, \hat{c}_N)$ is a $3N$-vector created from the measured values.

- The Likelihood of the data, given the set $\theta$ of parameters, is

$$
\mathcal{L}_{SN}(\hat{Z}|\theta) = |2\pi(\Sigma_d + A^T\Sigma_l A)|^{-1/2} \times \\
\exp[-(\hat{Z} - Y_0 A)(\Sigma_d + A^T\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^T/2],
$$

- $\Sigma_d$ is the covariance matrix of the data given by Betoule et al. 14
- $A$ is a $3N \times 3N$ block-diagonal matrix

$$
A = \begin{pmatrix}
1 & 0 & 0 \\
-\alpha & 1 & 0 \\
\beta & 0 & 1 \\
& & \ddots
\end{pmatrix}.
$$
Nielsen *et al.* 15 claimed a marginal evidence ($\lesssim 3\sigma$) for acceleration

Rubin and Hayden 2016

- criticised this approach as incomplete for the selection bias
- suggested using redshift dependent distributions
- presented the evidence for acceleration to be $\sim 4.2\sigma$.

Different methods for treating the selection bias in the SN data have been discussed in Kessler *et al.* 17.
Haridasu et al. 17 jointly analysed
- the JLA dataset with the Nielsen et al. statistical approach
- 31 OHD points from the dataset collected Farooq 2016.
- 6 BAO data from 6dFGS, SDSS DR7 and BOSS DR11
- 109 GRB observations: 0.1 < z < 8.1 (Amati et al. 2009)

... against low-redshift data
SN analysis agreement with Neilsen et al. 2015 (c.l. in grey levels)

The joint analysis improves the evidence for acceleration in the $k\Lambda$CDM model to $4.98\sigma$ (c.l. in violet levels)

<table>
<thead>
<tr>
<th>Data</th>
<th>$H_0$ $km\ s^{-1}/Mpc$</th>
<th>$\Omega_m$</th>
<th>$\Omega_\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JLA</td>
<td>-</td>
<td>0.34 ± 0.10</td>
<td>0.57 ± 0.15</td>
</tr>
<tr>
<td>Joint analysis</td>
<td>66.0 ± 2.0</td>
<td>0.35 ± 0.04</td>
<td>0.60 ± 0.08</td>
</tr>
</tbody>
</table>
For the SN analysis alone
- c.l. in grey levels
- the no-acceleration (dashed) line is at $4.56\sigma$

For the joint analysis
- c.l. in violet levels
- the no-acceleration (dashed) line is at $5.38\sigma$ (c.f. black point)

Consistency with $w = -1$

<table>
<thead>
<tr>
<th>Data</th>
<th>$H_0$ $\text{km s}^{-1}/\text{Mpc}$</th>
<th>$\Omega_m$ $\pm$ $\sigma$</th>
<th>$w$ $\pm$ $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JLA</td>
<td>-</td>
<td>$0.35 \pm 0.12$</td>
<td>$-0.92 \pm 0.30$</td>
</tr>
<tr>
<td>Joint analysis</td>
<td>$66.2 \pm 1.8$</td>
<td>$0.33 \pm 0.04$</td>
<td>$-0.90 \pm 0.12$</td>
</tr>
</tbody>
</table>
The (red) point $(\Omega_m, w) = (0, -1/3)$
- phenomenologically reproduces
  - the $R_h = ct$ cosmology

For the SN analysis alone
- c.l. in grey levels
  - this point is at $4.56\sigma$

For the joint analysis
- c.l. in violet levels
  - the no-acceleration (dashed) line is at $5.5\sigma$
Flat, power-law cosmological model

- In a power model $a(t) \propto t^n$ and $n > 1$ implies acceleration
- $R_h = ct$ coincides with the power-law model for $n = 1$
- The Friedman equation for the $\Lambda$CDM model

$$H(z)^2 = H_0^2 \left( \Omega_m (1 + z)^3 + \Omega_\Lambda (1 + z)^3(1+w) \right),$$

reduces to the functional form of a power-law model

$$H(z) = H_0 (1 + z)^{1/n}$$

for selected parameter values:

$$\Omega_m = 0 \quad \Omega_\Lambda = 1 \quad w = \frac{2 - 3n}{3n}$$
Power law cosmologies $\Omega_m = 0$, $\Omega_\Lambda = 1$, $w = (2 - 3n)/(3n)$

- The point $(\Omega_m, w) = (0, -0.38)$
  - corresponds to the best-fit value of $n = 1.08$ in power-law models
- For the joint analysis
  - c.l. in violet levels
  - this point is at $5\sigma$

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Best-fit parameters for the joint analysis with $1\sigma$ errors.

<table>
<thead>
<tr>
<th>Model</th>
<th>$H_0$ [km s$^{-1}$/Mpc]</th>
<th>$n$</th>
<th>$\Omega_m$</th>
<th>$r_d$ [Mpc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_h = ct$</td>
<td>62.4±1.4</td>
<td>1</td>
<td>-</td>
<td>148.3±3.6</td>
</tr>
<tr>
<td>Power-law</td>
<td>64.2±1.7</td>
<td>1.08±0.04</td>
<td>-</td>
<td>147.0±3.6</td>
</tr>
<tr>
<td>$\Lambda$CDM</td>
<td>66.4±1.8</td>
<td>-</td>
<td>0.361±0.023</td>
<td>148.6±3.7</td>
</tr>
</tbody>
</table>
Information analysis

- The Akaike Information criteria (Akaike 1974)
  - \( AIC = -2 \log \mathcal{L}^{\text{max}} + 2N_p \)
  - \( \Delta(AIC) \equiv AIC - AIC_{\text{ref}} \)
    - If \(< 0\): the model performs better than the reference one
    - If \(> 0\): the model performs worse than the reference one

- The Bayesian Information criteria (Schwarz 1978)
  - \( BIC = -2 \log \mathcal{L}^{\text{max}} + N_p \log(N_{\text{data}}) \)
  - \( \Delta(BIC) \equiv BIC - BIC_{\text{ref}} \)
    - If \(< 0\): the model performs better than the reference one
    - If \(> 0\): the model performs worse than the reference one

- \( N_p \) and \( N_{\text{data}} \) being the \# of parameters and data points

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Delta(AIC)_{\text{Joint}} )</th>
<th>( \Delta(AIC)_{\text{SN}} )</th>
<th>( \Delta(BIC)_{\text{Joint}} )</th>
<th>( \Delta(BIC)_{\text{SN}} )</th>
</tr>
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<tbody>
<tr>
<td>Power-law</td>
<td>28.02</td>
<td>2.0</td>
<td>28.02</td>
<td>2.0</td>
</tr>
<tr>
<td>( R_h = ct )</td>
<td>30.83</td>
<td>21.79</td>
<td>26.05</td>
<td>17.20</td>
</tr>
<tr>
<td>Milne</td>
<td>66.39</td>
<td>9.78</td>
<td>61.62</td>
<td>5.19</td>
</tr>
</tbody>
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Strong evidence for an accelerating universe
Luković et al. jointly analysed

- the JLA dataset with the Nielsen et al. statistical approach
- 23 OHD points from the dataset collected by Ding et al. 2015
- 6 BAO data from 6dFGS, SDSS DR7 and BOSS DR11

to test

- $k\Lambda$CDM cosmologies ($\Omega_k \neq 0$ and $w = -1$)
- $w\Lambda$CDM models ($\Omega_k = 0$ and $w \neq -1$)
- LTB models

against low-redshift data, in particular for the $H_0$ determination.
\( H_0 \) determination

<table>
<thead>
<tr>
<th>( H_0 )</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>( H_0 = (72 \pm 8) \text{ km s}^{-1}/\text{Mpc} )</td>
<td>HST Key Project</td>
</tr>
<tr>
<td>( H_0 = (73.0 \pm 1.8) \text{ km s}^{-1}/\text{Mpc} )</td>
<td>Riess16</td>
</tr>
<tr>
<td>( H_0 = (70.0 \pm 2.2) \text{ km s}^{-1}/\text{Mpc} )</td>
<td>WMAP9</td>
</tr>
<tr>
<td>( H_0 = (67.27 \pm 0.66) \text{ km s}^{-1}/\text{Mpc} )</td>
<td>Planck</td>
</tr>
</tbody>
</table>

- Indirect estimates < direct ones
- Riess16 and Planck in tension at the 3.0\( \sigma \) level
- New physics?
- Efstathiou14 reanalysed the Riess et al. 11 Cepheid data
  - \( H_0 = (72.5 \pm 2.5) \text{ km s}^{-1}/\text{Mpc} \), reducing the tension to 2\( \sigma \)
  - Concluded that there is no evidence for new physics.
ΛCDM best fit parameters: $H_0$

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<tr>
<th>Data</th>
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<td>JLA</td>
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<td>0.376 ± 0.031</td>
<td>0.624 ± 0.031</td>
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<tr>
<td>OHD</td>
<td>68.7 ± 3.3</td>
<td>0.319 ± 0.061</td>
<td>0.681 ± 0.061</td>
</tr>
<tr>
<td>BAO</td>
<td>67.3 ± 2.2</td>
<td>0.334 ± 0.042</td>
<td>0.689 ± 0.037</td>
</tr>
<tr>
<td>JLA+OHD</td>
<td>66.7 ± 2.0</td>
<td>0.366 ± 0.028</td>
<td>0.634 ± 0.028</td>
</tr>
<tr>
<td>JLA+OHD+BAO</td>
<td>67.8 ± 1.0</td>
<td>0.350 ± 0.016</td>
<td>0.650 ± 0.016</td>
</tr>
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The joint analysis is in excellent agreement with Planck, but 1.7σ and 2.5σ away from Efstathiou14 and Riess16.
ΛCDM best fit parameters: $\Omega_m$

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The values obtained from JLA only and JLA+OHD+BAO are 1.9σ away from Planck ($\Omega_m = 0.316 ± 0.009$).
### kΛCDM best fit parameters

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<td>0.569 ± 0.149</td>
</tr>
<tr>
<td>OHD</td>
<td>68.2 ± 5.7</td>
<td>0.291 ± 0.265</td>
<td>0.622 ± 0.539</td>
</tr>
<tr>
<td>BAO</td>
<td>68.7 ± 7.3</td>
<td>0.354 ± 0.106</td>
<td>0.646 ± 0.106</td>
</tr>
<tr>
<td>JLA+OHD</td>
<td>66.3 ± 2.2</td>
<td>0.319 ± 0.044</td>
<td>0.556 ± 0.144</td>
</tr>
<tr>
<td>JLA+OHD+BAO</td>
<td>68.1 ± 1.0</td>
<td>0.350 ± 0.016</td>
<td>0.650 ± 0.016</td>
</tr>
</tbody>
</table>

Nicola Vittorio

Strong evidence for an accelerating universe
### $wCDM$ best fit parameters

<table>
<thead>
<tr>
<th>Data</th>
<th>$H_0$ km s$^{-1}$/Mpc</th>
<th>$\Omega_m$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>JLA</td>
<td>-</td>
<td>0.347 ± 0.119</td>
<td>−0.92 ± 0.30</td>
</tr>
<tr>
<td>OHD</td>
<td>68.5 ± 7.2</td>
<td>0.318 ± 0.077</td>
<td>−0.98 ± 0.69</td>
</tr>
<tr>
<td>BAO</td>
<td>65.5 ± 8.0</td>
<td>0.329 ± 0.049</td>
<td>−0.93 ± 0.28</td>
</tr>
<tr>
<td>JLA+OHD</td>
<td>67.0 ± 1.9</td>
<td>0.318 ± 0.073</td>
<td>−0.86 ± 0.17</td>
</tr>
<tr>
<td>JLA+OHD+BAO</td>
<td>66.5 ± 1.8</td>
<td>0.346 ± 0.017</td>
<td>−0.93 ± 0.07</td>
</tr>
</tbody>
</table>

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Strong evidence for an accelerating universe
Lemaître-Tolman-Bondi models

- LTB metric

\[ ds^2 = c^2 dt^2 - \frac{R^2(r, t)}{1 - k(r)} dr^2 - R^2(r, t)[d\theta^2 + \sin^2 \theta \, d\phi^2] \]

- \( k(r) \) determines the spatial curvature of 3D space

- The analogous of the Friedmann equation

\[ H^2_{\perp}(r, t) = H^2_0(r) \left[ \frac{\Omega_m(r)}{a(r, t)^3} + \frac{\Omega_k(r)}{a(r, t)^2} \right] \]

- \( H_{\perp}(r, t) \equiv \dot{R}(r, t)/R(r, t); \quad H_0(r) \equiv H_{\perp}(r, t_0) \)
- \( \Omega_m(r) \equiv \frac{8\pi G \rho_m(r)}{3H^2_0(r)} \)
- \( \Omega_k(r) \equiv -k(r)c^2/[H^2_0(r)R^2_0(r)] \)
- For each shell, we have \( \Omega_m(r) + \Omega_k(r) \equiv 1. \)
For $\Omega_k(r) \geq 0$

\[
\begin{align*}
    a(r, t) &= \frac{\Omega_m(r)}{2\Omega_k(r)} (\cosh \eta - 1) \\
    t - t_B(r) &= \frac{1}{2H_0(r)} \frac{\Omega_m(r)}{\Omega_k^{3/2}(r)} (\sinh \eta - \eta)
\end{align*}
\]

We need an ansatz for the matter density profile

\[
\Omega_m(r) = \Omega_{\text{out}} - (\Omega_{\text{out}} - \Omega_{\text{in}}) e^{-r^2/2\rho^2}
\]
Along the radial null geodesic (Enqvist07)

\[
\frac{dr}{dz} = \frac{c \sqrt{1 - k(r)}}{(1 + z) \dot{R}'(r, t)} \\
\frac{dt}{dz} = -\frac{R'(r, t)}{(1 + z) \dot{R}'(r, t)}
\]

so that

\[
d_L(z) = (1 + z)^2 R[r(z), t(z)] = \frac{c}{H_0} (1 + z)^2 H_0 t_0 \tilde{R}
\]

\[
\tilde{R} = R[r(z), t(z)]/(ct_0)
\]

\[
H_0 t_0(0) = (H_0 \Omega_k^{3/2})^{-1} \left(\sqrt{\Omega_k} - \Omega_m \sinh^{-1} \sqrt{\Omega_k/\Omega_m}\right)
\]

The magnitude-redshift for the LTB model becomes

\[
m_B = \mathcal{M} - \alpha s + \beta c + 5 \log_{10} \left[ (1 + z)^2 H_0 t_0 \tilde{R}(z) \right] + 25
\]

\[
\mathcal{M} \equiv M_B^{\text{corr}} + 5 \log_{10} d_H
\]
For the $\rho$ best-fit value from the JLA analysis we get $R_0(r = \rho) \approx 2.5\text{ Gpc}/h$.

The $H_0$ best-fit value is low, $2.6\sigma$ away from Efstathiou14 and $3.4\sigma$ away from Riess16.
Information analysis

- The Akaike Information criteria (Akaike74)
  \[ AIC = -2 \log \mathcal{L}^{\text{max}} + 2N_p \]
  - \(N_p\) is the number of parameters
  - \(\Delta(AIC) \equiv AIC - AIC_{\text{ref}}\)
  - If \(< 0\): the model performs better than the reference one
  - If \(> 0\): the model performs better than the reference one

- The Bayes factor
  \[ K = \frac{\mathcal{L}_{r}^{\text{max}}}{\mathcal{L}^{\text{max}}} \] (Jeffreys85)
  - "Odds" for the model against the reference model
  - \(< 0.1\): strong evidence against the reference model
  - \(> 10\): strong evidence against the model

<table>
<thead>
<tr>
<th>Model</th>
<th>(\Delta(AIC))</th>
<th>(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Lambda)CDM</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(k\Lambda)CDM</td>
<td>1.69</td>
<td>0.85</td>
</tr>
<tr>
<td>(w)CDM</td>
<td>1.40</td>
<td>0.74</td>
</tr>
<tr>
<td>LTB</td>
<td>9.41</td>
<td>40</td>
</tr>
</tbody>
</table>
Data analysis

Strong evidence for an accelerating universe
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Conclusions

- The current expansion of our universe is definitely accelerated

\( \Lambda \text{CDM} \)
- is still the most consistent model with the observations
- it constitutes the best base line for a concordance model in cosmology.

Open issues for \( \Lambda \text{CDM} \)
- What is \( \Lambda \)?
- Why \( \Omega_\Lambda \approx \Omega_m \)?
- Why \( \Lambda \) is so small?
- Why model-dependent, indirect estimates of \( H_0 \) are not consistent with the direct measurements?

....