Cutoffs, Curvature and Quantum Noise

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How much can the sound

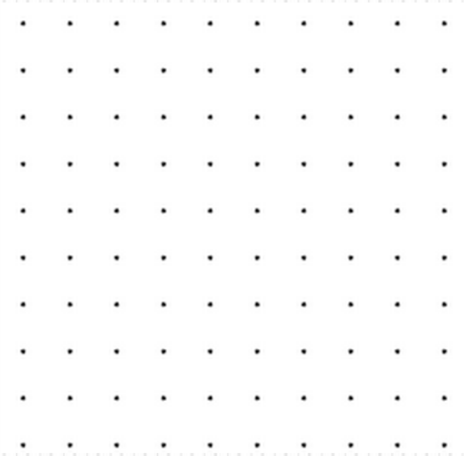
tell about the shape ?

How much can quantum vacuum noise tell about spacetime curvature?

How could this work?

- First in Minkowski space:

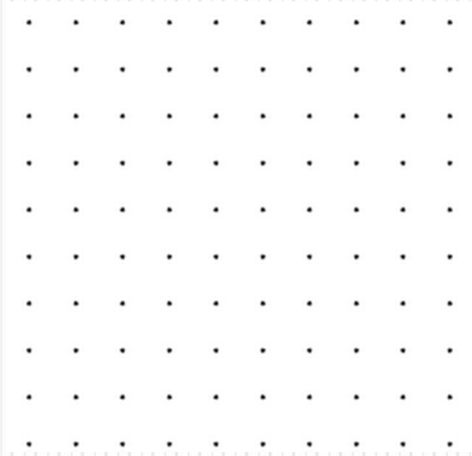
$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi$$



- It's a system of coupled harmonic oscillators
- Their quantum fluctuations are correlated via the Laplacian
- Quantified by: 2-point functions such as the propagator

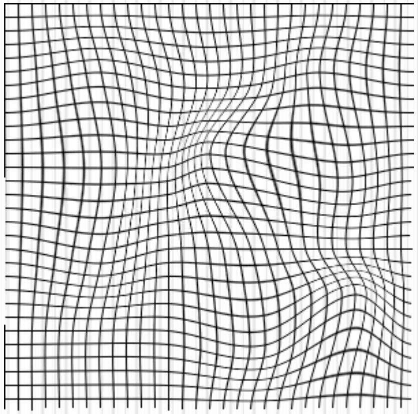
Remarks

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\psi$$



- Area law for entanglement entropy / holography
- Entanglement can be harvested from the vacuum
- Quantum collect calling
- Quantum energy teleportation

In curved spacetime



- Klein Gordon equation now:

$$\left(\frac{1}{\sqrt{g}}\partial_\mu \sqrt{g} g^{\mu\nu}\partial_\nu + \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

- Curvature affects the coupling of the oscillators

➔Curvature affects the correlations of quantum noise

➔Curvature affects the propagator

How much does the propagator know about the curvature?

Result

- For dimensions D>2, the metric can be expressed in terms of the free Feynman propagator of a Klein Gordon field:

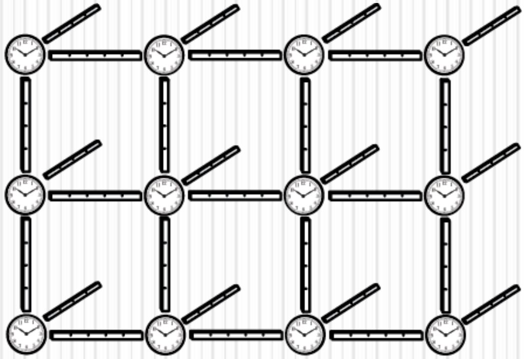
$$g_{ij}(y) = -\frac{1}{2}\left[\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right]^{\frac{2}{D-2}} \lim_{x\to y} \frac{\partial}{\partial x^i}\frac{\partial}{\partial y^j}\left(G(x,y)^{\frac{2}{2-D}}\right)$$

- Proof: covariance, geodesic coordinates, asymptotic behavior
- Also in paper: worked-out examples

➔ In principle: Can replace the metric with the propagator!

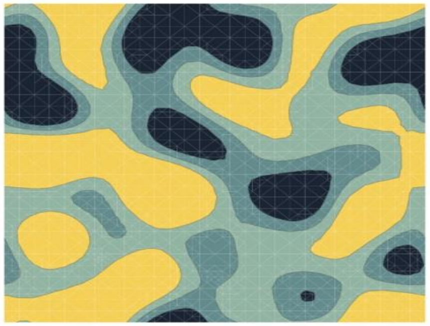
Remark

- Einstein built relativity on rods and clocks



- But no rods and clocks at sub-atomic scales!

- Instead: as distance proxy, can use strength of correlators:



Remark

Is the spectrum of the quantum noise sufficient?

Remark

At least in euclidean signature case, yes:

- Given the spectra of scalar, vector and tensor noise

$\Rightarrow$ One can reconstruct the manifold in infinitesimal steps.

(We had to get spectral geometry unstuck by introducing perturbation theory)

Remark

Spectral geometry,

i.e., the study of sound from shape,

could provide a bridge between the mathematical languages of GR and QT.

Enter Planck scale physics

- Within a QFT, moving from low energies to high energies,

  how does Planck scale physics effectively first manifest itself?

- What kind of a natural UV cutoff?

- Assuming covariance, little choice but cutoff on spectrum of d'Alembertian.

Remarks

- Not a lattice!

  (Only if required periodicity in momentum space)

- Require Dirichlet BC ➔
  - Covariantly generalized minimum length uncertainty principle
  - Get Shannon sampling theory:
    - Spacetime is discrete & continuous, in the same way that information is.

Covariant natural UV cutoff

Cut off of the spectrum of the d'Alembertian:

$$Z[J] = \int_F e^{iS[\phi]+i\int J\phi \, d^n x} D[\phi]$$

The space of fields, F, in the QFT path integral

is spanned by the eigenfunctions w. eigenvalues:

$$|\lambda_i| < \Lambda$$

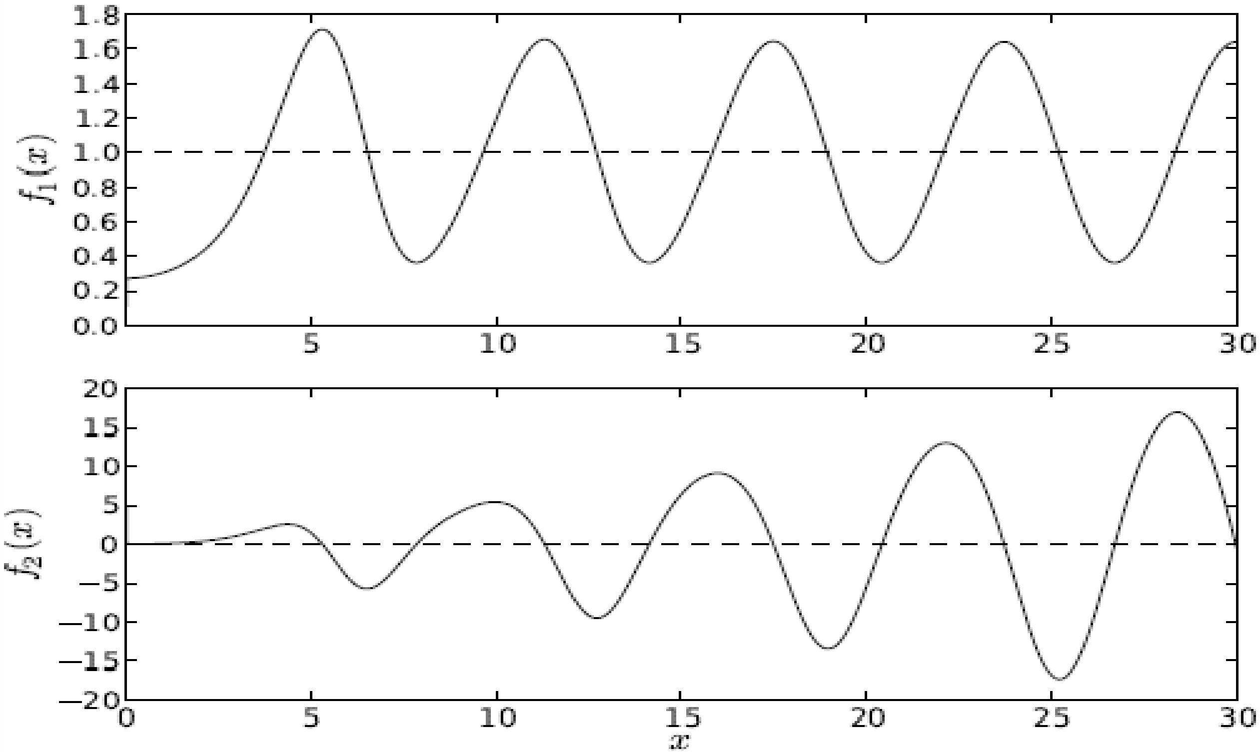
Apply the covariant cutoff to:

$$g_{ij}^{\Lambda}(y) \equiv -\frac{1}{2}\left[\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right]^{\frac{2}{D-2}} \lim_{x\to y} \frac{\partial}{\partial x^i}\frac{\partial}{\partial y^j} G_{\Lambda}(x,y)^{\frac{2}{2-D}}$$

- Notice: the UV limits x $\longrightarrow$ y and $\Lambda \longrightarrow \infty$ compete !
- Do they commute?
- It's more subtle with UV cutoff: they do only "on average"

Example D=3 flat euclidean space:

$$g_{\alpha\beta}(x,y) = \delta_{\alpha\beta} f_1(\Lambda r_{xy}) + \frac{(x_\alpha - y_\alpha)(x_\beta - y_\beta)}{r_{xy}^2} f_2(\Lambda r_{xy})$$



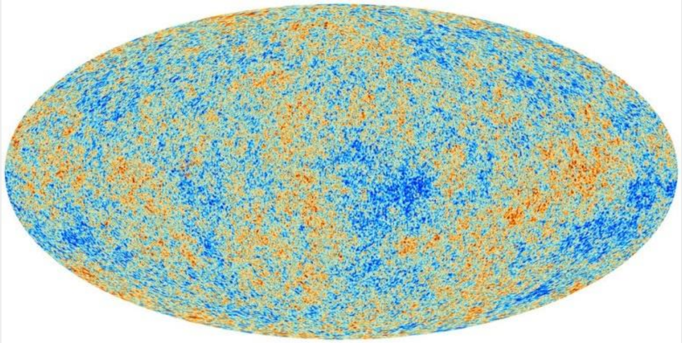
- **Oscillations. We recover usual metric "on average"**

This is not testable at the LHC

- How could one possibly test if such a natural UV cutoff exists or not?

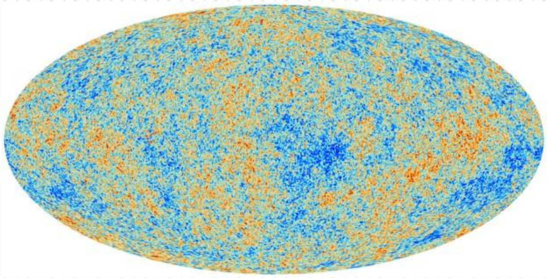
Any signature visible in the CMB ?

CMB's structure originated close to Planck scale



Hubble scale in inflation may have been only about 5 orders from the Planck scale.

Natural UV cutoffs in inflation



Multiple groups have non-covariant predictions for CMB.

- No agreement if the effect is first or second order in (Planck length/Hubble length), i.e. could be say O(10^-5) or O(10^-10)

Challenge, therefore:

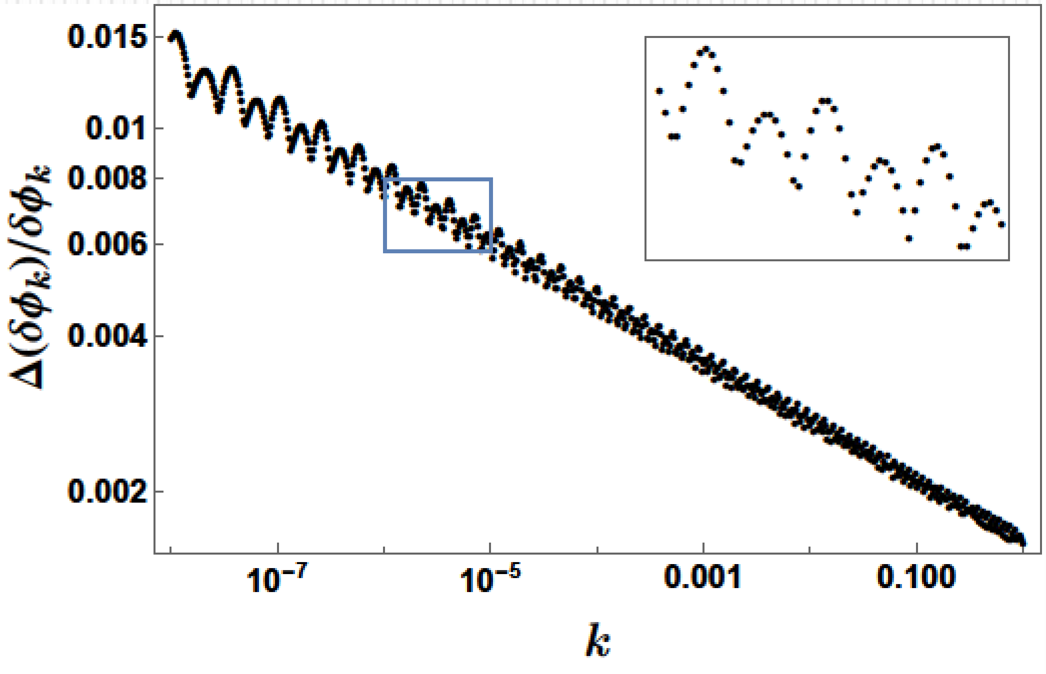
Calculate predictions with local Lorentz covariant UV cutoff

New PRL with:

Aidan Chatwin-Davies (CalTech) and Robert Martin (U. Cape Town)

Results for sharp covariant cutoff

- Power law inflation: relative change in (tensor) spectrum



- These characteristic oscillations' amplitude is linear in (Planck length/Hubble length)

Conclusions

- **Does the quantum noise know all about the spacetime curvature?**

  Yes, already the free scalar propagator does.

  (And in the euclidean case, certain spectra suffice too.)

- **Also:**

  Via inflation, propagators may conceivably indicate the onset of Planck scale physics.