In collaboration with
Ki-Seok Kim(POSTECH) and Chanyoung Park(APCTP, POSTECH)

Emergent Metric from Kitaev Superconductor Model [a]

Miok Park

Institute for Basic Science(IBS-CALDES), Pohang, S. Korea

Lemaitre workshop at Vatican Observartory
“Black Holes, Gravitational Waves and Spacetime Singularities”

May 11, 2017

[a] 1610.07312

Motivation

- **High Energy Physics View**

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Holographic Superconductors”, JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

  - Black Hole Thermodynamics

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

  - Black Hole Thermodynamics
  - RN-AdS/Van-deer Waals gas at critical point, [1]

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Holographic Superconductors”, JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

- Black Hole Thermodynamics
- RN-AdS/Van-deer Waals gas at critical point, [1]
- Holography(AdS/CMT e.g. holographic superconductor) [2]

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Holographic Superconductors”, JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

  - Black Hole Thermodynamics
  - RN-AdS/Van-deer Waals gas at critical point, [1]
  - Holography(AdS/CMT e.g. holographic superconductor) [2]
  - **Can we construct a spacetime from many body system?**

- **Condensed Matter Physics View**

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

  - Black Hole Thermodynamics
  - RN-AdS/Van-deer Waals gas at critical point, [1]
  - Holography(AdS/CMT e.g. holographic superconductor) [2]
  - **Can we construct a spacetime from many body system?**

- **Condensed Matter Physics View**

many strongly correlated system to study

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Motivation

- **High Energy Physics View**

We have a fundamental question :

What is a nature of spacetime?

  - Black Hole Thermodynamics
  - RN-AdS/Van-deer Waals gas at critical point, [1]
  - Holography(AdS/CMT e.g. holographic superconductor) [2]
  - **Can we construct a spacetime from many body system?**

- **Condensed Matter Physics View**

many strongly correlated system to study

"Could it provide a new tool to investigate those systems?"

[1] D. Kubiznak and R. B. Mann, "P-V criticality of charged AdS black holes", JHEP 1207, 033 (2012)

[2] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, "Holographic Superconductors", JHEP 0812, 015 (2008)

Our Approach
from a transeverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Our Approach

from a transeverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Why a transverse-Ising model?

Our Approach

from a transeverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Why a transverse-Ising model?

- Simple (no interacting term), but describe the superconductors

Our Approach

from a transeverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Why a transverse-Ising model?

- Simple (no interacting term), but describe the superconductors
- which is known as Kitaev superconductor model

Our Approach

from a transeverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Why a transverse-Ising model?

- Simple (no interacting term), but describe the superconductors
- which is known as Kitaev superconductor model
- Extract metric structure from Kitaev superconductor model by performing the RG process, where RG solution for a coupling parameter induces the background spacetime to be curved.

Process

Process

1. Map the transverse-field Ising model to Kitaev Superconductor Model
2. Do real-space Renormalisation Group
3. Obtain RG equation for coupling parameter
4. Identify the number of repetition of RG to extra radial direction
5. **Plug RG solution for coupling parameter to the partition function**
6. Dirac equation in curved spacetime in two dimensions
7. **Embed two dimensional spacetime to three dimensions**
8. Compare 5. and 7.

- **The transverse-field Ising model**

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + \lambda\sigma_i^x\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

,

Kitaev Superconductor Model

Topological Phase Transition

- **The transverse-field Ising model**

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + \lambda\sigma_i^x\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field. This Hamiltonian enjoys Z_2 symmetry and shows a quantum phase transition at zero temperature

$$\langle\sigma_i^z\rangle \neq 0 \qquad \text{ferromagnetic phase for } \lambda < \lambda_c$$
$$\langle\sigma_i^z\rangle = 0 \qquad \text{paramagnetic phase for } \lambda > \lambda_c$$

- **The transverse-field Ising model**

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + \lambda\sigma_i^x\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j}(c_i^\dagger + c_i) \tag{1}$$

where c_i is a spinless fermion field.

,

- **The transverse-field Ising model**

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + \lambda\sigma_i^x\right) = -\frac{1}{2}\sum_{i=1}^{N}\left[J\lambda(2c_i^\dagger c_i - 1) + J(c_i^\dagger c_{i+1} + c_{i+1}c_i + h.c.)\right]$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j}(c_i^\dagger + c_i) \tag{1}$$

where c_i is a spinless fermion field.

,

- **The transverse-field Ising model**

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + \lambda\sigma_i^x\right) = -\frac{1}{2}\sum_{i=1}^{N}\left[J\lambda(2c_i^\dagger c_i - 1) + J(c_i^\dagger c_{i+1} + c_{i+1}c_i + h.c.)\right]$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j}(c_i^\dagger + c_i) \tag{1}$$

where c_i is a spinless fermion field.

- **Majorana Decomposition**

$$\gamma_i = c_i + c_i^\dagger, \qquad \tilde{\gamma}_i = i(c_i - c_i^\dagger) \tag{2}$$

,

- **The transverse-field Ising model**

$$H = 2i\left(\sum_{i=1}^{N} t_1 \gamma_i \tilde{\gamma}_i + t_2 \tilde{\gamma}_i \gamma_{i+1}\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j} (c_i^\dagger + c_i) \tag{1}$$

where c_i is a spinless fermion field.

- **Majorana Decomposition**

$$\gamma_i = c_i + c_i^\dagger, \qquad \tilde{\gamma}_i = i(c_i - c_i^\dagger) \tag{2}$$

,

Kitaev Superconductor Model

Topological Phase Transition

- **The transverse-field Ising model**

$$H = 2i\left(\sum_{i=1}^{N} t_1 \gamma_i \tilde{\gamma}_i + t_2 \tilde{\gamma}_i \gamma_{i+1}\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j}(c_i^\dagger + c_i) \tag{1}$$

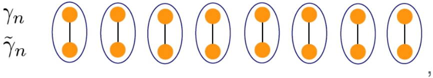
where c_i is a spinless fermion field.

- **Majorana Decomposition**

$$\gamma_i = c_i + c_i^\dagger, \qquad \tilde{\gamma}_i = i(c_i - c_i^\dagger) \tag{2}$$

- **Strong pairing phase trivial/Weak pairing phase topological superconductor**

$$\mathcal{H} \propto \sum_n i\tilde{\gamma}_n \gamma_n$$



Kitaev Superconductor Model

Topological Phase Transition

- **The transverse-field Ising model**

$$H = 2i\left(\sum_{i=1}^{N} t_1\gamma_i\tilde{\gamma}_i + t_2\tilde{\gamma}_i\gamma_{i+1}\right)$$

where J is a ferromagnetic coupling constant and λ is a transverse magnetic field.

- **Jordan-Wigner transformation**

$$\sigma_i^x = 2c_i^\dagger c_i - 1, \qquad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j}(c_i^\dagger + c_i) \qquad (1)$$

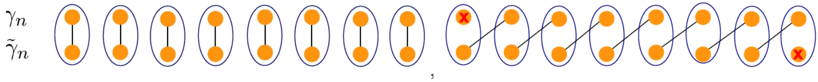
where c_i is a spinless fermion field.

- **Majorana Decomposition**

$$\gamma_i = c_i + c_i^\dagger, \qquad \tilde{\gamma}_i = i(c_i - c_i^\dagger) \qquad (2)$$

- **Strong pairing phase trivial/Weak pairing phase topological superconductor**

$$\mathcal{H} \propto \sum_n i\tilde{\gamma}_n\gamma_n \qquad\qquad \mathcal{H} \propto \sum_n i\tilde{\gamma}_n\gamma_{n+1}$$



Real-Space Renormalisation

- **Employ Nambu-spinor representation**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$\psi_i = \begin{pmatrix} c_i \\ c_i^\dagger \end{pmatrix}$$

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$\psi_i = \begin{pmatrix} c_i \\ c_i^\dagger \end{pmatrix}$$

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[-\int_0^\beta d\tau \sum_{i=1}^{N} \left\{\psi_i^\dagger \left(\partial_\tau I + J\lambda\tau_3\right)\psi_i - J\psi_i^\dagger \left(\tau_3 - i\tau_2\right)\psi_{i+1}\right\}\right]$$

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[-\int_0^{\beta} d\tau \sum_{i=1}^{N} \left\{ \psi_i^{\dagger} \Big(\partial_\tau I + J\lambda\tau_3 \Big) \psi_i - J\psi_i^{\dagger} \Big(\tau_3 - i\tau_2 \Big) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

$$Z_{k-1} = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\Big(-i\omega I + J\lambda\tau_3 \Big) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp\left[-\sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \Big(-i\omega I + J\lambda\tau_3 \Big) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \Big(\tau_3 - i\tau_2 \Big) \psi_i^{(k-1)} \right\} \right]$$

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[- \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \Big(\partial_\tau I + J\lambda\tau_3 \Big) \psi_i - J\psi_i^\dagger \Big(\tau_3 - i\tau_2 \Big) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

$$Z_{k-1} = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\Big(-i\omega I + J\lambda\tau_3 \Big) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp\left[- \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \Big(-i\omega I + J\lambda\tau_3 \Big) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \Big(\tau_3 - i\tau_2 \Big) \psi_i^{(k-1)} \right\} \right]$$

1. **separate the site index into even and odd**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[- \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \Big(\partial_\tau I + J\lambda\tau_3 \Big) \psi_i - J\psi_i^\dagger \Big(\tau_3 - i\tau_2 \Big) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

$$Z_{k-1} = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\Big(-i\omega I + J\lambda\tau_3 \Big) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp\left[- \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \Big(-i\omega I + J\lambda\tau_3 \Big) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \Big(\tau_3 - i\tau_2 \Big) \psi_i^{(k-1)} \right\} \right]$$

1. **separate the site index into even and odd**
2. **perform the gaussian integration for odd-site**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[- \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left(\partial_\tau I + J\lambda\tau_3 \right) \psi_i - J\psi_i^\dagger \left(\tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

$$Z_{k-1} = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\left(-i\omega I + J\lambda\tau_3 \right) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp\left[- \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \left(-i\omega I + J\lambda\tau_3 \right) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \left(\tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right]$$

1. **separate the site index into even and odd**
2. **perform the gaussian integration for odd-site**
3. **rescale the fermion field to reproduce the original lattice**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[-\int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left(\partial_\tau I + J\lambda\tau_3 \right) \psi_i - J\psi_i^\dagger \left(\tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

1. **separate the site index into even and odd**

$$\begin{aligned} Z_{k-1} = &\exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\left(-i\omega I + J\lambda\tau_3 \right) \right\} \int \Pi_{i=1}^{N/2} D\psi_{i+}^{(k-1)} D\psi_{i-}^{(k-1)} \\ &\exp\Bigg[-\sum_{i\omega} \sum_{i=1}^{N/2} \Big\{ \psi_{i+}^{(k-1)\dagger} \left(-i\omega I + J\lambda\tau_3 \right) \psi_{i+}^{(k-1)} + \psi_{i-}^{(k-1)\dagger} \left(-i\omega I + J\lambda\tau_3 \right) \psi_{i-}^{(k-1)} \\ &- J_{k-1} \psi_{i-}^{(k-1)\dagger} \left(\tau_3 - i\tau_2 \right) \psi_{i+}^{(k-1)} - J_{k-1} \psi_{i+1+}^{(k-1)\dagger} \left(\tau_3 - i\tau_2 \right) \psi_{i-}^{(k-1)} \Big\} \Bigg] \end{aligned} \tag{3}$$

2. **perform the gaussian integration for odd-site**
3. **rescale the fermion field to reproduce the original lattice**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[- \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left(\partial_\tau I + J\lambda\tau_3 \right) \psi_i - J\psi_i^\dagger \left(\tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

1. **separate the site index into even and odd**
2. **perform the gaussian integration for odd-site**

$$Z_k = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln\left(-i\omega I + J\lambda\tau_3 \right) + \sum_{i\omega} \sum_{i=1}^{N/2} \ln\left(-i\omega I + J\lambda\tau_3 \right) \right\}$$

$$\int \Pi_{i=1}^{N/2} D\psi_{i+}^{(k-1)} \exp\left[- \sum_{i\omega} \sum_{i=1}^{N/2} \left\{ \psi_{i+}^{(k-1)\dagger} \left(-i\omega I + J\lambda\tau_3 \right) \psi_{i+}^{(k-1)} \right.\right.$$

$$\left.\left. - \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_{i+1+}^{(k-1)\dagger} \left(\tau_3 - i\tau_2 \right) \psi_{i+}^{(k-1)} \right\} \right]$$

3. **rescale the fermion field to reproduce the original lattice**

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[- \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left(\partial_\tau I + J\lambda\tau_3 \right) \psi_i - J\psi_i^\dagger \left(\tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

1. **separate the site index into even and odd**
2. **perform the gaussian integration for odd-site**
3. **rescale the fermion field to reproduce the original lattice**

$$Z_k = \exp\left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k}{2} \ln\left(-i\omega I + J\lambda\tau_3 \right) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k)} \exp\left[- \sum_{i\omega} \sum_{i=1}^{N} \right.$$
$$\left. \left\{ \psi_i^{(k)\dagger} \left(-i\omega I + J\lambda\tau_3 \right) \psi_i^{(k)} - \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_{i+1}^{(k)\dagger} \left(\tau_3 - i\tau_2 \right) \psi_i^{(k)} \right\} \right]$$

Real-Space Renormalisation

- **Employ Nambu-spinor representation**

$$Z = \int \Pi_{i=1}^{N} D\psi_i \exp\left[-\int_0^{\beta} d\tau \sum_{i=1}^{N} \left\{ \psi_i^{\dagger}\Big(\partial_\tau I + J\lambda\tau_3\Big)\psi_i - J\psi_i^{\dagger}\Big(\tau_3 - i\tau_2\Big)\psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the $(k-1)^{th}$ RG tr.

$$Z_{k-1} = \exp\left\{ \sum_{i\omega}\sum_{i=1}^{N} \frac{k-1}{2} \ln\Big(-i\omega I + J\lambda\tau_3\Big) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp\left[-\sum_{i\omega}\sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger}\Big(-i\omega I + J\lambda\tau_3\Big)\psi_i^{(k-1)} - J_{k-1}\psi_{i+1}^{(k-1)\dagger}\Big(\tau_3 - i\tau_2\Big)\psi_i^{(k-1)} \right\} \right]$$

1. **separate the site index into even and odd**
2. **perform the gaussian integration for odd-site**
3. **rescale the fermion field to reproduce the original lattice**

$$Z_k = \exp\left\{ \sum_{i\omega}\sum_{i=1}^{N} \frac{k}{2} \ln\Big(-i\omega I + J\lambda\tau_3\Big) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k)} \exp\left[-\sum_{i\omega}\sum_{i=1}^{N} \right.$$

$$\left. \left\{ \psi_i^{(k)\dagger}\Big(-i\omega I + J\lambda\tau_3\Big)\psi_i^{(k)} - \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_{i+1}^{(k)\dagger}\Big(\tau_3 - i\tau_2\Big)\psi_i^{(k)} \right\} \right]$$

RG equation for coupling parameter
and extra radial direction

- **RG equation becomes**

RG equation for coupling parameter
and extra radial direction

- **RG equation becomes**

$$J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2,$$

RG equation for coupling parameter
and extra radial direction

- **RG equation becomes**

$$J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2,$$

$$\frac{J_k - J_{k-1}}{k-(k-1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2$$

RG equation for coupling parameter
and extra radial direction

- **RG equation becomes**

$$J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2,$$

$$\frac{J_k - J_{k-1}}{k - (k-1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2$$

- **Taking** $k - 1 \to \frac{z}{a}$ **and** $k \to \frac{z+dz}{a}$**,**
(where a is an ultraviolet (UV) energy scale, set to be $a = 1$ for simplicity)

$$\frac{dJ(i\omega, r)}{dr} = -J(i\omega, r) + \frac{2J\lambda}{(J\lambda)^2 + \omega^2} [J(i\omega, r)]^2 \qquad (3)$$

RG equation for coupling parameter
and extra radial direction

- **RG equation becomes**

$$J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2,$$

$$\frac{J_k - J_{k-1}}{k-(k-1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2$$

- **Taking $k-1 \to \frac{z}{a}$ and $k \to \frac{z+dz}{a}$,**
(where a is an ultraviolet (UV) energy scale, set to be $a=1$ for simplicity)

$$\frac{dJ(i\omega, r)}{dr} = -J(i\omega, r) + \frac{2J\lambda}{(J\lambda)^2 + \omega^2}[J(i\omega, r)]^2 \qquad (3)$$

- **IR action turns to**

$$\mathcal{S}_{IR}(\Lambda) \approx \int_0^\beta d\tau \int_0^L dx \psi^\dagger(x, \tau, \Lambda) \Big\{ \partial_\tau I + J\Big(\frac{\lambda[1-(\lambda/2)]e^\Lambda}{[1-(\lambda/2)]e^\Lambda - 1} - \frac{\lambda/2}{1-[1-(\lambda/2)]e^\Lambda} \partial_x^2 \Big) \tau_3 + \frac{J\lambda}{1-[1-(\lambda/2)]e^\Lambda}(-i\partial_x)\tau_1 \Big\} \psi(x, \tau, \Lambda)$$

Dirac equation in 2D curved spacetime
embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \quad (4)$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \quad (5)$$

Dirac equation in 2D curved spacetime
embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \tag{4}$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \tag{5}$$

- **compare above with the previous action** S_{IR}

Dirac equation in 2D curved spacetime
embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \quad (4)$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \quad (5)$$

- **compare above with the previous action** $S_{IR} \Rightarrow \mathcal{A}$ and $\mathcal{B}$ are determined

Dirac equation in 2D curved spacetime

embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \quad (4)$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \quad (5)$$

- **compare above with the previous action** $S_{IR} \Rightarrow \mathcal{A}$ and $\mathcal{B}$ are determined
- **embedding this into three dimensional spacetime**

$$ds^2 = -e^{-2\mathcal{A}}dt^2 + e^{-2\mathcal{B}}dx^2 + e^{-2\mathcal{C}}dr^2 \quad (6)$$

Dirac equation in 2D curved spacetime
embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \quad (4)$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \quad (5)$$

- **compare above with the previous action** $S_{IR} \Rightarrow \mathcal{A}$ and $\mathcal{B}$ are determined
- **embedding this into three dimensional spacetime**

$$ds^2 = -e^{-2\mathcal{A}}dt^2 + e^{-2\mathcal{B}}dx^2 + e^{-2\mathcal{C}}dr^2 \quad (6)$$

- **consider a following choice :** $\sqrt{-\det g(\Lambda)} = \sqrt{e^{-2(\mathcal{A}+\mathcal{B}+\mathcal{C})}} = e^{-\mathcal{Y}}$

Dirac equation in 2D curved spacetime
embedded in three dimensions

- **two dimensional Dirac equation in the curved spacetime as follows**

$$ds^2 = -e^{-2\mathcal{A}(x)}dt^2 + e^{-2\mathcal{B}(x)}dx^2, \quad (4)$$

$$\left(i\partial_t + \frac{i}{2}\mathcal{A}'(x)e^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x + ie^{\mathcal{B}(x)-\mathcal{A}(x)}\sigma_x\partial_x - e^{-\mathcal{A}(x)}\sigma_z m\right)\Psi = 0 \quad (5)$$

- **compare above with the previous action** $S_{IR} \Rightarrow \mathcal{A}$ and $\mathcal{B}$ are determined
- **embedding this into three dimensional spacetime**

$$ds^2 = -e^{-2\mathcal{A}}dt^2 + e^{-2\mathcal{B}}dx^2 + e^{-2\mathcal{C}}dr^2 \quad (6)$$

- **consider a following choice :** $\sqrt{-\det g(\Lambda)} = \sqrt{e^{-2(\mathcal{A}+\mathcal{B}+\mathcal{C})}} = e^{-\mathcal{Y}}$

$$g_{tt} = \frac{J^2(\lambda-2)^2}{m^2}\left(1 - \frac{2}{\lambda} + \frac{2}{\lambda}e^{-\Lambda}\right)^{-2}, \quad (7)$$

$$g_{xx} = \frac{(\lambda-2)^2}{m^2}e^{2\Lambda}, \quad (8)$$

$$g_{rr} = \frac{4m^4}{J^2(\lambda-2)^4}\left(1 - \frac{2}{\lambda} + \frac{2}{\lambda}e^{-\Lambda}\right)^2 e^{-2\Lambda-2\mathcal{Y}}. \quad (9)$$

Hereafter, we take $e^\Lambda = r$ and $\mathcal{Y} = -2\Lambda$

Metric Structures
for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

,

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \qquad (10)$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$),

,

Metric Structures
for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \qquad (10)$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda-2)^2 \qquad (11)$$

,

Metric Structures
for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda-2)^2 \tag{11}$$

Taking the limit of $\lambda \to \lambda_c = 2$

,

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda-2)^2 \tag{11}$$

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = -\frac{r^2}{4\mathcal{F}^2}dt'^2 + \frac{r^2}{4\mathcal{F}^2}dx'^2 + \frac{4\mathcal{F}^2}{r^2}dr^2 \tag{12}$$

,

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{(|\lambda'|+\frac{2}{r})^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{(|\lambda'|+\frac{2}{r})^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda-2)^2 \tag{11}$$

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = -\frac{r^2}{4\mathcal{F}^2}dt'^2 + \frac{r^2}{4\mathcal{F}^2}dx'^2 + \frac{4\mathcal{F}^2}{r^2}dr^2 \tag{12}$$

- **non-topological($\lambda' > 0$) and topological($\lambda' < 0$) superconducting phase**

,

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{(|\lambda'|+\frac{2}{r})^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{(|\lambda'|+\frac{2}{r})^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda - 2)^2 \tag{11}$$

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = -\frac{r^2}{4\mathcal{F}^2}dt'^2 + \frac{r^2}{4\mathcal{F}^2}dx'^2 + \frac{4\mathcal{F}^2}{r^2}dr^2 \tag{12}$$

- **non-topological**($\lambda' > 0$) **and topological**($\lambda' < 0$) **superconducting phase**

$$(r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2dx'^2 + dr'^2,$$

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \tag{10}$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda - 2)^2 \tag{11}$$

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = -\frac{r^2}{4\mathcal{F}^2}dt'^2 + \frac{r^2}{4\mathcal{F}^2}dx'^2 + \frac{4\mathcal{F}^2}{r^2}dr^2 \tag{12}$$

- **non-topological($\lambda' > 0$) and topological($\lambda' < 0$) superconducting phase**

$$(r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2dx'^2 + dr'^2,$$

$$R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} = \frac{6a^4\lambda'^8\left(\lambda'^2r^2+2\right)}{m^8(|\lambda'|r+2)^8}$$

Metric Structures

for critical point and two superconducting phases

Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2\lambda'^2}{m^2}\frac{(|\lambda'|+2)^2}{\left(|\lambda'|+\frac{2}{r}\right)^2}dt^2 + \frac{\lambda'^2}{m^2}r^2dx^2 + \frac{4m^4}{J^2\lambda'^4}\frac{\left(|\lambda'|+\frac{2}{r}\right)^2}{(|\lambda'|+2)^2}dr^2 \qquad (10)$$

- **at quantum critical point** ($\lambda = \lambda_c = 2$ or $\lambda' = 0$), natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = \mathcal{F}J(\lambda-2)^2 \qquad (11)$$

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = -\frac{r^2}{4\mathcal{F}^2}dt'^2 + \frac{r^2}{4\mathcal{F}^2}dx'^2 + \frac{4\mathcal{F}^2}{r^2}dr^2 \qquad (12)$$

- **non-topological($\lambda' > 0$) and topological($\lambda' < 0$) superconducting phase**

$$(r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2dx'^2 + dr'^2,$$

$$R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} = \frac{6a^4\lambda'^8\left(\lambda'^2r^2+2\right)}{m^8(|\lambda'|r+2)^8} \sim \infty \ \text{at} \ r = \frac{2}{\lambda'}$$

Einstein Equation

Since the metric is asymptotically flat, it should satisfy

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu} \tag{13}$$

and we found

$$T_{tt} = -\frac{J^4(|\lambda'|+2)^4 \lambda'^6 r^2}{2m^6(|\lambda'|r+2)^5} \quad (\sim 0, \quad r \to \infty) \tag{14}$$

$$T_{rr} = \frac{2}{r^2(|\lambda'|r+2)} \quad (\sim 0, \quad r \to \infty), \tag{15}$$

$$T_{xx} = \frac{J^2(|\lambda'|+2)^2(1-r|\lambda'|)\lambda'^6 r^2}{m^6(|\lambda'|r+2)^4} \quad (\sim 0, \quad r \to \infty) \tag{16}$$

Summary and Future Work

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**
- **emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition**

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**
- **emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition**

- **Is there a natural way to have a constraint on radial component?**

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**
- **emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition**

- **Is there a natural way to have a constraint on radial component?**
- **How to characterize the topological property for the spacetime having a naked singularity which is $\lambda < 2$ case?**

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**
- **emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition**

- **Is there a natural way to have a constraint on radial component?**
- **How to characterize the topological property for the spacetime having a naked singularity which is $\lambda < 2$ case?**
- **dictionaries between a field theory and a bulk?**

Summary and Future Work

- **we constructed an emergent spacetime from the Kitaev superconductor model**
- **emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition**

- **Is there a natural way to have a constraint on radial component?**
- **How to characterize the topological property for the spacetime having a naked singularity which is $\lambda < 2$ case?**
- **dictionaries between a field theory and a bulk?**
- **We are also working on O(N) vector model, Kondo model..**