In collaboration with
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Emergent Metric from Kitaev Superconductor Model

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“Black Holes, Gravitational Waves and Spacetime Singularities”

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\[a1610.07312\]
Motivation

- High Energy Physics View

- Condensed Matter Physics View

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Motivation

- **High Energy Physics View**

  We have a fundamental question:

- **Condensed Matter Physics View**

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Motivation

High Energy Physics View

We have a fundamental question:

What is a nature of spacetime?

Condensed Matter Physics View

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1 D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 1207, 033 (2012)
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▶ Black Hole Thermodynamics

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We have a fundamental question:
What is a nature of spacetime?

- Black Hole Thermodynamics
- RN-AdS/Van-deer Waals gas at critical point, \(^1\)

Condensed Matter Physics View

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▶ High Energy Physics View

We have a fundamental question:
What is a nature of spacetime?

▶ Black Hole Thermodynamics
▶ RN-AdS/Van-deer Waals gas at critical point,¹
▶ Holography(AdS/CMT e.g. holographic superconductor)²

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High Energy Physics View

We have a fundamental question:
What is the nature of spacetime?

- Black Hole Thermodynamics
- RN-AdS/Van-deer Waals gas at critical point,
- Holography (AdS/CMT e.g. holographic superconductor)
- Can we construct a spacetime from many body system?

Condensed Matter Physics View

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➡️ Black Hole Thermodynamics
➡️ RN-AdS/Van-deer Waals gas at critical point, \(^1\)
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many strongly correlated system to study

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Motivation

High Energy Physics View

- We have a fundamental question: What is the nature of spacetime?
- Black Hole Thermodynamics
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Condensed Matter Physics View

Many strongly correlated system to study

“Could it provide a new tool to investigate these systems?”

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1 D. Kubiznak and R. B. Mann, “P-V criticality of charged AdS black holes”, JHEP 1207, 033 (2012)
Our Approach
from a transverse-Ising model to an emergent spacetime

D dimensional field theory $\Rightarrow$ (D+1) dimensional bulk spacetime
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We start with a transverse-Ising model.
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D dimensional field theory ⇒ (D+1) dimensional bulk spacetime

We start with a transverse-Ising model.

Why a transverse-Ising model?

- Simple (no interacting term), but describe the superconductors
- which is known as Kitaev superconductor model
- Extract metric structure from Kitaev superconductor model by performing the RG process, where RG solution for a coupling parameter induces the background spacetime to be curved.
Process

1. Map the transverse-field Ising model to Kitaev Superconductor Model
2. Do real-space Renormalisation Group
3. Obtain RG equation for coupling parameter
4. Identify the number of repetition of RG to extra radial direction
5. Plug RG solution for coupling parameter to the partition function
6. Dirac equation in curved spacetime in two dimensions
7. Embed two dimensional spacetime to three dimensions
Process

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The transverse-field Ising model

\[ H = -\frac{J}{2} \sum_{i=1}^{N} \left( \sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x \right) \]

where \( J \) is a ferromagnetic coupling constant and \( \lambda \) is a transverse magnetic field.
The transverse-field Ising model

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where \( J \) is a ferromagnetic coupling constant and \( \lambda \) is a transverse magnetic field. This Hamiltonian enjoys \( \mathbb{Z}_2 \) symmetry and shows a quantum phase transition at zero temperature:

- \( \langle \sigma_i^z \rangle \neq 0 \) ferromagnetic phase for \( \lambda < \lambda_c \)
- \( \langle \sigma_i^z \rangle = 0 \) paramagnetic phase for \( \lambda > \lambda_c \)
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Jordan-Wigner transformation

\[ \sigma_i^x = 2c_i^\dagger c_i - 1, \quad \sigma_i^z = (-1)^{i-1} e^{\pm i\pi \sum_{j=1}^{i-1} c_j^\dagger c_j} (c_i^\dagger + c_i) \]

where \( c_i \) is a spinless fermion field.
The transverse-field Ising model

\[ H = -\frac{J}{2} \sum_{i=1}^{N} (\sigma_i^z \sigma_{i+1}^z + \lambda \sigma_i^x) = -\frac{1}{2} \sum_{i=1}^{N} \left[ J\lambda(2c_i^\dagger c_i - 1) + J(c_i^\dagger c_{i+1} + c_{i+1} c_i + h.c.) \right] \]

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where \( c_i \) is a spinless fermion field.

Majorana Decomposition

\[ \gamma_i = c_i + c_i^\dagger, \quad \tilde{\gamma}_i = i(c_i - c_i^\dagger) \]  \hspace{1cm} (2)
The transverse-field Ising model

\[ H = 2i \left( \sum_{i=1}^{N} t_1 \gamma_i \tilde{\gamma}_i + t_2 \tilde{\gamma}_i \gamma_{i+1} \right) \]

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Strong pairing phase trivial/Weak pairing phase topological superconductor
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Majorana Decomposition

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Strong pairing phase trivial/Weak pairing phase topological superconductor
Real-Space Renormalisation

- Employ Nambu-spinor representation
Employ Nambu-spinor representation

\[ \psi_i = \begin{pmatrix} c_i \\ c_i^\dagger \end{pmatrix} \]
We begin with the effective renormalised partition function after the $(k-1)$th RG tr.

1. separate the site index into even and odd
2. perform the gaussian integration for odd-site
3. rescale the fermion field to reproduce the original lattice

Employ Nambu-spinor representation

$$
\psi_i = \begin{pmatrix}
c_i \\
c_i^\dagger
\end{pmatrix}
$$

$$
Z = \int \prod_{i=1}^N D\psi_i \exp \left[ - \int_0^\beta d\tau \sum_{i=1}^N \left\{ \psi_i^\dagger \left( \partial_\tau I + J \lambda \tau_3 \right) \psi_i - J \psi_i^\dagger \left( \tau_3 - i \tau_2 \right) \psi_{i+1} \right\} \right]
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Real-Space Renormalisation

Employ Nambu-spinor representation

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We begin with the effective renormalised partition function after the \((k - 1)^{th}\) RG tr.

\[ Z_{k-1} = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln \left( -i\omega I + J\lambda \tau_3 \right) \right\} \int \prod_{i=1}^{N} D\psi_i^{(k-1)} \]

\[ \exp \left[ - \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \left( -i\omega I + J\lambda \tau_3 \right) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right] \]
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\[
\exp \left[ - \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)^\dagger} \left( - i\omega I + J\lambda \tau_3 \right) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)^\dagger} \left( \tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right]
\]

1. separate the site index into even and odd
Real-Space Renormalisation

Employ Nambu-spinor representation

\[ Z = \int \Pi_{i=1}^{N} D\psi_i \exp \left[ - \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left( \partial_\tau I + J_3 \tau_3 \right) \psi_i - J_3 \psi_i^\dagger \left( \tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right] \]

We begin with the effective renormalised partition function after the \((k - 1)\)th RG tr.

\[ Z_{k-1} = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k - 1}{2} \ln \left( - i\omega I + J_3 \tau_3 \right) \right\} \int \Pi_{i=1}^{N} D\psi_i^{(k-1)} \]

\[ \exp \left[ - \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)}^\dagger \left( - i\omega I + J_3 \tau_3 \right) \psi_i^{(k-1)} - J_{k-1} \psi_i^{(k-1)}^\dagger \left( \tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right] \]

1. separate the site index into even and odd
2. perform the gaussian integration for odd-site
Real-Space Renormalisation

- **Employ Nambu-spinor representation**

\[
Z = \int \prod_{i=1}^{N} D\psi_i \exp \left[ -\int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left( \partial_\tau - iJ_3 \right) \psi_i - J\psi_i^\dagger \left( \tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]
\]

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Z_{k-1} = \exp \left\{ \sum_{i}\sum_{\omega} \sum_{i=1}^{N} \frac{k-1}{2} \ln \left( -i\omega - iJ_3 \right) \right\} \int \prod_{i=1}^{N} D\psi_i^{(k-1)}
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\exp \left[ -\sum_{i}\sum_{\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \left( -i\omega - iJ_3 \right) \psi_i^{(k-1)} - J_{k-1}\psi_{i+1}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right]
\]

1. separate the site index into even and odd
2. perform the gaussian integration for odd-site
3. rescale the fermion field to reproduce the original lattice
Real-Space Renormalisation

- Employ Nambu-spinor representation

\[ Z = \int \prod_{i=1}^{N} D\psi_i \exp \left[ - \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left( \partial_\tau I + J\lambda\tau_3 \right) \psi_i - J\psi_i^\dagger \left( \tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right] \]

We begin with the effective renormalised partition function after the \((k - 1)^{th}\) RG tr.

1. separate the site index into even and odd

\[ Z_{k-1} = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k - 1}{2} \ln \left( -i\omega l + J\lambda\tau_3 \right) \right\} \int \prod_{i=1}^{N/2} D\psi_{i+}^{(k-1)} D\psi_{i-}^{(k-1)} \]

\[ \exp \left[ - \sum_{i\omega} \sum_{i=1}^{N/2} \left\{ \psi_{i+}^{(k-1)\dagger} \left( -i\omega l + J\lambda\tau_3 \right) \psi_{i+}^{(k-1)} + \psi_{i-}^{(k-1)\dagger} \left( -i\omega l + J\lambda\tau_3 \right) \psi_{i-}^{(k-1)} \right. \right. \]

\[ \left. \left. - J_{k-1} \psi_{i-}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_{i+}^{(k-1)} - J_{k-1} \psi_{i+1+}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_{i-}^{(k-1)} \right\} \right] \quad (3) \]

2. perform the gaussian integration for odd-site

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\[
Z = \int \prod_{i=1}^{N} D\psi_i \exp \left[ - \int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left( \partial_\tau I + J\lambda \tau_3 \right) \psi_i - J\psi_i^\dagger \left( \tau_3 - i\tau_2 \right) \psi_{i+1} \right\} \right]
\]

We begin with the effective renormalised partition function after the \((k - 1)^{th}\) RG tr.

1. separate the site index into even and odd
2. perform the gaussian integration for odd-site

\[
Z_k = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k - 1}{2} \ln \left( -i\omega I + J\lambda \tau_3 \right) + \sum_{i\omega} \sum_{i=1}^{N/2} \ln \left( -i\omega I + J\lambda \tau_3 \right) \right\}
\]

\[
\int \prod_{i=1}^{N/2} D\psi_{i+}^{(k-1)} \exp \left[ - \sum_{i\omega} \sum_{i=1}^{N/2} \left\{ \psi_{i+}^{(k-1)\dagger} \left( -i\omega I + J\lambda \tau_3 \right) \psi_{i+}^{(k-1)} \right. \right.
\]

\[
- \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_{i+1+}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_{i+}^{(k-1)} \right\] \]

3. rescale the fermion field to reproduce the original lattice
Real-Space Renormalisation

Employ Nambu-spinor representation

\[ Z = \int \prod_{i=1}^{N} D\psi_i \exp \left[ - \int_{0}^{\beta} d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger \left( \partial_\tau I + J \lambda \tau_3 \right) \psi_i - J \psi_i^\dagger \left( \tau_3 - i \tau_2 \right) \psi_{i+1} \right\} \right] \]

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\[ Z_k = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k}{2} \ln \left( -i\omega I + J \lambda \tau_3 \right) \right\} \int \prod_{i=1}^{N} D\psi_i^{(k)} \exp \left[ - \sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k)}^\dagger \left( -i\omega I + J \lambda \tau_3 \right) \psi_i^{(k)} - \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_i^{(k)}^\dagger \left( \tau_3 - i \tau_2 \right) \psi_i^{(k)} \right\} \right] \]
Real-Space Renormalisation

Employ Nambu-spinor representation

$$Z = \int \prod_{i=1}^{N} D\psi_i \exp \left[ -\int_0^\beta d\tau \sum_{i=1}^{N} \left\{ \psi_i^\dagger (\partial_\tau I + J\lambda \tau_3) \psi_i - J\psi_i^\dagger (\tau_3 - i\tau_2) \psi_{i+1} \right\} \right]$$

We begin with the effective renormalised partition function after the \((k - 1)^{th}\) RG tr.

$$Z_{k-1} = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k - 1}{2} \ln \left( -i\omega I + J\lambda \tau_3 \right) \right\} \int \prod_{i=1}^{N} D\psi_i^{(k-1)}$$

$$\exp \left[ -\sum_{i\omega} \sum_{i=1}^{N} \left\{ \psi_i^{(k-1)\dagger} \left( -i\omega I + J\lambda \tau_3 \right) \psi_i^{(k-1)} - J_{k-1} \psi_{i+1}^{(k-1)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_i^{(k-1)} \right\} \right]$$

1. separate the site index into even and odd
2. perform the gaussian integration for odd-site
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$$Z_k = \exp \left\{ \sum_{i\omega} \sum_{i=1}^{N} \frac{k}{2} \ln \left( -i\omega I + J\lambda \tau_3 \right) \right\} \int \prod_{i=1}^{N} D\psi_i^{(k)} \exp \left[ -\sum_{i\omega} \sum_{i=1}^{N} \right. $$

$$\left\{ \psi_i^{(k)\dagger} \left( -i\omega I + J\lambda \tau_3 \right) \psi_i^{(k)} - \frac{-2J_{k-1}^2 J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} \psi_{i+1}^{(k)\dagger} \left( \tau_3 - i\tau_2 \right) \psi_i^{(k)} \right\} \right]$$
RG equation for coupling parameter and extra radial direction

RG equation becomes

\[ \frac{J_k}{J_{k-1}} = -2J_{\lambda}(\omega - J_{\lambda})(\omega + J_{\lambda})J_2^2, \]

\[ J_k - J_{k-1} = -J_{k-1} + -2J_{\lambda}(\omega - J_{\lambda})(\omega + J_{\lambda})J_2^2. \]

Taking \( k-1 \rightarrow z_a \) and \( k \rightarrow z + dz_a \), (where \( a \) is an ultraviolet (UV) energy scale, set to be \( a = 1 \) for simplicity)

\[ \int dJ(\omega, r) = -J(\omega, r) + 2J_{\lambda}(J_{\lambda}^2 + \omega^2) \]

\[ J(\omega, r) \]

IR action turns to

\[ S^{\text{IR}}(\Lambda) \approx \int \beta_0 d\tau \int L_0 dx \psi^\dagger(x, \tau, \Lambda) \{ \partial_\tau I + J(\lambda) \[ 1 - \left( \frac{\lambda}{2} \right) \] e^{\Lambda} \[ 1 - \left( \frac{\lambda}{2} \right) \] e^{\Lambda - 1 - \lambda/2} \]

\[ - \lambda/2 \] \[ \partial_2 x \] \[ \tau^3 \] + \[ 1 - \left( \frac{\lambda}{2} \right) \] \[ e^{\Lambda} \] (\[ -i \partial x \]) \[ \tau \] \[ 1 - \left( \frac{\lambda}{2} \right) \] \[ e^{\Lambda} \]

\[ \psi(x, \tau, \Lambda) \]
RG equation for coupling parameter and extra radial direction

- RG equation becomes

\[ J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2, \]
RG equation for coupling parameter and extra radial direction

RG equation becomes

\[
J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2,
\]

\[
\frac{J_k - J_{k-1}}{k - (k - 1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2
\]
RG equation for coupling parameter and extra radial direction

RG equation becomes

\[ J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2, \]

\[ \frac{J_k - J_{k-1}}{k - (k-1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2 \]

Taking \( k - 1 \rightarrow \frac{z}{a} \) and \( k \rightarrow \frac{z + dz}{a} \),

(where \( a \) is an ultraviolet (UV) energy scale, set to be \( a = 1 \) for simplicity)

\[ \frac{dJ(i\omega, r)}{dr} = -J(i\omega, r) + \frac{2J\lambda}{(J\lambda)^2 + \omega^2} [J(i\omega, r)]^2 \]  \( (3) \)
RG equation for coupling parameter and extra radial direction

\[ J_k = \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2, \]

\[ \frac{J_k - J_{k-1}}{k - (k - 1)} = -J_{k-1} + \frac{-2J\lambda}{(-i\omega + J\lambda)(-i\omega - J\lambda)} J_{k-1}^2 \]

Taking \( k - 1 \to \frac{z}{a} \) and \( k \to \frac{z + dz}{a} \),

(\text{where } a \text{ is an ultraviolet (UV) energy scale, set to be } a = 1 \text{ for simplicity})

\[
\frac{dJ(i\omega, r)}{dr} = -J(i\omega, r) + \frac{2J\lambda}{(J\lambda)^2 + \omega^2 [J(i\omega, r)]^2} \quad (3)
\]

IR action turns to

\[
S_{IR}(\Lambda) \approx \int_0^\beta d\tau \int_0^L dx \psi^\dagger(x, \tau, \Lambda) \left\{ \partial_\tau I + J \left( \frac{\lambda[1 - (\lambda/2)]e^\Lambda}{[1 - (\lambda/2)]e^\Lambda - 1} \right) \psi \right\}
\]

\[
- \frac{\lambda/2}{1 - [1 - (\lambda/2)]e^\Lambda \partial_x^2} \tau_3 + \frac{J\lambda}{1 - [1 - (\lambda/2)]e^\Lambda} (-i\partial_x) \tau_1 \psi(x, \tau, \Lambda)
\]
two dimensional Dirac equation in the curved spacetime as follows

\[ ds^2 = -e^{-2A(x)} dt^2 + e^{-2B(x)} dx^2, \]  

\[ \left( i\partial_t + \frac{i}{2} A'(x)e^{B(x)-A(x)}\sigma_x + ie^{B(x)-A(x)}\sigma_x \partial_x - e^{-A(x)}\sigma_z m \right) \psi = 0 \]
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compare above with the previous action \( S_{IR} \)
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embedding this into three dimensional spacetime
\[ ds^2 = -e^{-2A}dt^2 + e^{-2B}dx^2 + e^{-2C}dr^2 \]
two dimensional Dirac equation in the curved spacetime as follows
\[
\begin{align*}
&ds^2 = -e^{-2A(x)} dt^2 + e^{-2B(x)} dx^2, \\
&(i\partial_t + \frac{i}{2} A'(x) e^{B(x)-A(x)} \sigma_x + i e^{B(x)-A(x)} \sigma_x \partial_x - e^{-A(x)} \sigma_z m) \psi = 0
\end{align*}
\]

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\[
ds^2 = -e^{-2A} dt^2 + e^{-2B} dx^2 + e^{-2C} dr^2
\]

consider a following choice : \( \sqrt{-\det g(\Lambda)} = \sqrt{e^{-2(\Lambda+B+C)}} = e^{-\gamma} \)
two dimensional Dirac equation in the curved spacetime as follows
\[ ds^2 = -e^{-2A(x)} dt^2 + e^{-2B(x)} dx^2, \] (4)
\[ \left( i\partial_t + \frac{i}{2} A'(x) e^{B(x) - A(x)} \sigma_x + ie^{B(x) - A(x)} \sigma_x \partial_x - e^{-A(x)} \sigma_z m \right) \psi = 0 \] (5)

compare above with the previous action \( S_{IR} \Rightarrow A \) and \( B \) are determined

embedding this into three dimensional spacetime
\[ ds^2 = -e^{-2A} dt^2 + e^{-2B} dx^2 + e^{-2C} dr^2 \] (6)

consider a following choice : \( \sqrt{-\det g(\Lambda)} = \sqrt{e^{-2(A+B+C)}} = e^{-\mathcal{Y}} \)
\[ g_{tt} = \frac{J^2(\lambda - 2)^2}{m^2} \left( 1 - \frac{2}{\lambda} + \frac{2}{\lambda} e^{-\Lambda} \right)^{-2}, \] (7)
\[ g_{xx} = \frac{(\lambda - 2)^2}{m^2} e^{2\Lambda}, \] (8)
\[ g_{rr} = \frac{4m^4}{J^2(\lambda - 2)^4} \left( 1 - \frac{2}{\lambda} + \frac{2}{\lambda} e^{-\Lambda} \right)^2 e^{-2\Lambda-2\mathcal{Y}}. \] (9)

Hereafter, we take \( e^\Lambda = r \) and \( \mathcal{Y} = -2\Lambda \)
Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2 \lambda^2}{m^2} \frac{(|\lambda'| + 2)^2}{(|\lambda'| + \frac{2}{r})^2} dt^2 + \frac{\lambda^2}{m^2} r^2 dx^2 + \frac{4m^4}{J^2 \lambda^4} \frac{(|\lambda'| + \frac{2}{r})^2}{(|\lambda'| + 2)^2} dr^2$$

(10)
Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2 \lambda'^2}{m^2} \left( \frac{|\lambda'| + 2}{\lambda'} \right)^2 dt^2 + \frac{\lambda'^2}{m^2} r^2 dx^2 + \frac{4m^4}{J^2 \lambda'^4} \left( \frac{|\lambda'| + 2}{r} \right)^2 dr^2$$  \hspace{1cm} (10)

- at quantum critical point $(\lambda = \lambda_c = 2 \text{ or } \lambda' = 0)$,
Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

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- at quantum critical point ($\lambda = \lambda_c = 2$ or $\lambda' = 0$),
  natural to assume $\frac{m^4}{J^2 (\lambda - 2)^2} \sim 1$, and so we set
  $$m^2 = \mathcal{F} J (\lambda - 2)^2 \quad (11)$$
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$$ds^2 = -\frac{r^2}{4\mathcal{F}^2} dt'^2 + \frac{r^2}{4\mathcal{F}^2} dx'^2 + \frac{4\mathcal{F}^{-2}}{r^2} dr^2 \quad (12)$$
Replacing \( \lambda - 2 = |\lambda'| \) so that \( \lambda - 2 = \lambda' \) for \( \lambda > 2 \) and \( \lambda - 2 = -\lambda' \) for \( \lambda < 2 \)

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ds^2 = -\frac{J^2 \lambda'^2}{m^2} \frac{(|\lambda'| + 2)^2}{(|\lambda'| + \frac{2}{r})^2} dt^2 + \frac{\lambda'^2}{m^2} r^2 dx^2 + \frac{4m^4}{J^2 \lambda'^4} \frac{(|\lambda'| + \frac{2}{r})^2}{(|\lambda'| + 2)^2} dr^2
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- **non-topological**(\( \lambda' > 0 \)) and **topological**(\( \lambda' < 0 \)) superconducting phase
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- **non-topological ($\lambda' > 0$) and topological ($\lambda' < 0$) superconducting phase**

\[ (r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2 dx'^2 + dr'^2, \]
Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2 \lambda'^2}{m^2} \frac{(|\lambda'| + 2)^2}{(|\lambda'| + \frac{2}{r})^2} dt'^2 + \frac{\lambda'^2}{m^2} r^2 dx'^2 + \frac{4m^4}{J^2 \lambda'^4} \frac{(|\lambda'| + \frac{2}{r})^2}{(|\lambda'| + 2)^2} dr'^2$$

(10)

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(12)

- **non-topological**($\lambda' > 0$) and **topological**($\lambda' < 0$) superconducting phase

  $$(r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2 dx'^2 + dr'^2,$$

  $$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} = \frac{6a^4 \lambda'^8 (\lambda'^2 r^2 + 2)}{m^8(|\lambda'|r + 2)^8}$$
Replacing $\lambda - 2 = |\lambda'|$ so that $\lambda - 2 = \lambda'$ for $\lambda > 2$ and $\lambda - 2 = -\lambda'$ for $\lambda < 2$

$$ds^2 = -\frac{J^2 \lambda'^2}{m^2} \frac{(|\lambda'| + 2)^2}{(|\lambda'| + \frac{2}{r})^2} dt^2 + \frac{\lambda'^2}{m^2} r^2 dx^2 + \frac{4m^4}{J^2 \lambda'^4} \frac{(|\lambda'| + \frac{2}{r})^2}{(|\lambda'| + 2)^2} dr^2$$ \hspace{1cm} (10)

\[ \textbf{at quantum critical point} \ (\lambda = \lambda_c = 2 \text{ or } \lambda' = 0), \]

natural to assume $\frac{m^4}{J^2(\lambda-2)^2} \sim 1$, and so we set

$$m^2 = F J (\lambda - 2)^2$$ \hspace{1cm} (11)

Taking the limit of $\lambda \to \lambda_c = 2$

$$ds^2 = - \frac{r^2}{4F^2} dt'^2 + \frac{r^2}{4F^2} dx'^2 + \frac{4F^{-2}}{r^2} dr^2$$ \hspace{1cm} (12)

\[ \textbf{non-topological}(\lambda' > 0) \text{ and topological}(\lambda' < 0) \text{ superconducting phase} \]

$$(r \to \infty) \quad ds^2 \approx -dt'^2 + r'^2 dx'^2 + dr'^2,$$

$$R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} = \frac{6a^4 \lambda'^8 (\lambda'^2 r^2 + 2)}{m^8 (|\lambda'| r + 2)^8} \sim \infty \text{ at } r = \frac{2}{\lambda'}$$
Since the metric is asymptotically flat, it should satisfy

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$  \hspace{1cm} (13)$$

and we found

$$T_{tt} = - \frac{J^4(|\lambda'| + 2)^4 \lambda'^6 r^2}{2m^6(|\lambda'|r + 2)^5} \quad (\sim 0, \quad r \to \infty)$$  \hspace{1cm} (14)$$

$$T_{rr} = \frac{2}{r^2(|\lambda'|r + 2)} \quad (\sim 0, \quad r \to \infty),$$  \hspace{1cm} (15)$$

$$T_{xx} = \frac{J^2(|\lambda'| + 2)^2(1 - r|\lambda'|)\lambda'^6 r^2}{m^6(|\lambda'|r + 2)^4} \quad (\sim 0, \quad r \to \infty)$$  \hspace{1cm} (16)$$
Summary and Future Work

▶ We constructed an emergent spacetime from the Kitaev superconductor model.
▶ An emergent spacetime description for the Kitaev model could be an order parameter for topological phase transition.
▶ Is there a natural way to have a constraint on radial component?
▶ How to characterize the topological property for the spacetime having a naked singularity which is $\lambda < \frac{2}{3}$ case?
▶ Dictionaries between a field theory and a bulk?
▶ We are also working on O(N) vector model, Kondo model, etc.
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