Cosmic Censorship in Quantum Einstein Gravity

A. Bonanno, B. Koch, A. Platania, Class. Quantum Grav. 34 (2017) 095012

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Gravitational collapse and singularities

- **Singularity theorems**: Spacetime singularities are a general feature of General Relativity

- The gravitational collapse of a sufficiently massive star gives rise to a **gravitational singularity**

- Is the singularity always covered by an **event horizon**?

- Can **naked singularities** arise from gravitational collapse?

**Cosmic Censorship Conjecture (CCC) - Penrose (1969)**
The generic singularities arising in the gravitational collapse are always covered by an event horizon.
Classical Gravitational Collapse: Vaidya space-time

Spherically Symmetric collapsing object

\[ ds^2 = -f(r, v) \, dv^2 + 2 \, dv \, dr + r^2 \, d\Omega^2 \]

\[ f(r, v) = 1 - \frac{2 \, G_0 \, m(v)}{r} \quad \text{\textit{m(v) is a "mass function"}} \]

- Describes the spacetime around an object with variable mass
- Corresponds to an exact solution of Einstein field equations with

\[ T_{\mu\nu} = \rho(r, v) \, \partial_\mu v \, \partial_\nu v \quad \text{\textit{Type II fluid}} \]

\[ \rho(r, v) \equiv \frac{\dot{m}(v)}{4\pi r^2} \]
Parametrization of the mass function

\[ m(v) = \begin{cases} 
0 & v < 0 \\
\lambda v & 0 \leq v < \bar{v} \\
\bar{m} & v \geq \bar{v}
\end{cases} \]

Critical case: \( \lambda \leq \frac{1}{16 G_0} \)

A. Papapetrou, A random walk in relativity and cosmology.
Hindustan Publishing Co., New Delhi, India (1985)

Outgoing radial null geodesics

\[
\frac{|r(v) - \mu_- v|^{-\mu_-}}{|r(v) - \mu_+ v|^{-\mu_+}} = \tilde{C}
\]

\[ \mu_{\pm} = \frac{1 \pm \sqrt{1 - 16 \lambda G_0}}{4} \]

Classical models for the gravitational collapse predict the formation of naked singularities

\textit{Israel, Canadian Journal of Physics, 1986, 64(2): 120-127}
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Asymptotic Safety in Quantum Gravity

Problem:
General Relativity is not (perturbatively) renormalizable

=> Possible solution (Weinberg, 1976):
Generalized notion of renormalizability

Functional Renormalization Group
FRG combines functional methods with the Wilsonian idea of Renormalization Group

- Evolution of the action in the theory space
  \[ \frac{k}{\beta} \lambda_k \Gamma_k = \frac{1}{2} \text{Tr} \left( \frac{k}{\beta} R_k \right) \]  
  Wettermich equation (1993)

- RG flow: connects Physics of different scales k

The microscopic theory is identified by the UV fixed points of \( \beta \) functions

\[ k \beta_k \lambda_k(k) \equiv \beta_k(\lambda_k^1, \lambda_k^2, \ldots) = 0 \]

Gaussian fixed point (GFP) -> Asymptotic Freedom
Non-Gaussian fixed point (NGFP) -> Asymptotic Safety
Functional Renormalization Group

FRG combines functional methods with the Wilsonian idea of Renormalization Group

- Evolution of the action in the theory space

\[ k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left[ \frac{k \partial_k \mathcal{R}_k}{\Gamma_k^{(2)} + \mathcal{R}_k} \right] \]

Wetterich equation (1993)

- **RG flow**: connects Physics of different scales \( k \)

The microscopic theory is identified by the **UV fixed points** of \( \beta \) functions

\[ k \partial_k \lambda_i(k) \equiv \beta_{\lambda_i}(\lambda_1^*, \lambda_2^*, \ldots) = 0 \]

**Gaussian fixed point (GFP)** ---\> Asymptotic Freedom
**Non-Gaussian fixed point (NGFP)** ---\> Asymptotic Safety
Quantum Einstein Gravity

Einstein-Hilbert truncation:

\[
\Gamma(k) = \frac{1}{16\pi G(k)} \int \sqrt{-g} \left\{ -R + 2\Lambda(k) \right\} + S_{gf} + S_{gh}
\]

- **Gaussian fixed point** \( g^* = 0 \) and \( \lambda^* = 0 \) (free theory, saddle point);

- **Non-Gaussian fixed point** \( g^* > 0 \) and \( \lambda^* > 0 \) (UV attractive);

\[
G(k) = g(k) k^2 \\
\Lambda(k) = \lambda(k) k^{-2}
\]
Classical Vaidya space-time

\[ ds^2 = -f_c(r, v) \, dv^2 + 2 \, dv \, dr + r^2 \, d\Omega^2 \]

\[ f_c(r, v) = 1 - \frac{2 \, m(v) \, G_0}{r} \]

Basic idea:
Include the leading quantum effects by "RG-improving" the classical Vaidya solution

\[ f_c(r, v) \rightarrow f_q(r, v) = 1 - \frac{2 \, m(v)}{r} \frac{G_0}{1 + \omega \, G_0 \, [k(r)]^2} \]

Running Newton’s constant

\[ G(k) = \frac{G_0}{1 + (G_0/g_*) \, k^2} \]

Cutoff identification

We identify $k(r)$ with the energy density of a null free falling observer

$$k(r) \equiv \xi \frac{4}{\lambda} \sqrt{\rho(r,v)} = \xi \sqrt{\frac{m(v)}{4\pi r^2}}$$

Bonanno, Esposito, Rubano, Scudellaro. Class. Quant. Grav. 23 (2006) 3103

RG-improved lapse function

$$f_k(r,v) = 1 - \frac{2 G_0 m(v)}{r + \alpha \sqrt{\lambda}}, \quad \alpha = \frac{\xi^2 G_0}{\sqrt{4\pi g_*}}$$

This is a "generalized Vaidya spacetime", solution of Einstein equation with a mixture of radiation and a new effective fluid with

$$\sigma(r) = \frac{\lambda \bar{v}}{4\pi r^2} \frac{\alpha \sqrt{\lambda}}{(r + \alpha \sqrt{\lambda})^2}, \quad p(r) = \frac{\lambda \bar{v}}{4\pi r} \frac{\alpha \sqrt{\lambda}}{(r + \alpha \sqrt{\lambda})^3}$$

The effect of a running Newton's constant is to produce a shift in the radial coordinate $r(v)$
General solution for the outgoing radial null geodesics

\[
\log \left[ 2\lambda G_0 v^2 - (r(v) + \alpha \sqrt{\lambda})v + 2(r(v) + \alpha \sqrt{\lambda})^2 \right] + \frac{-2 \arctan \left( \frac{v - 4[r(v) + \alpha \sqrt{\lambda}]}{v \sqrt{-1 + 16 \lambda G_0}} \right)}{\sqrt{-1 + 16 \lambda G_0}} = C
\]

- The critical value of the radiation rate has to be determined numerically.
- Because of the "quantum shift" the critical value is >1/16G

\[ \lambda \leq \lambda_c \quad \text{Naked Singularity} \]
"On the nature of singularities in General Relativity"

- In the singularity theorems nothing is specified on the “nature” of the singularities

- The physical relevance of a space-time singularity is determined by its strength

- **Strong curvature singularities**: characterized by the divergence of the gravitational tidal forces

- **Integrable singularities**: the integrated tidal forces are finite and the spacetime may be extended through the singularity
Gravitational collapse in generalized Vaidya spacetimes and singularity strength


Let us consider the generalized Vaidya spacetime

\[ ds^2 = -f(r, v) \, dv^2 + 2 \, dv \, dr + r^2 \, d\Omega^2 \]

\[ f(r, v) = 1 - \frac{2 \, M(r, v)}{r} \]

Geodesic equation for null rays

\[ \frac{dv}{dr} = 2 \left(1 - \frac{2 \, M(r, v)}{r}\right)^{-1} \]

\[ \begin{aligned}
\frac{dv(t)}{dt} &= 2 \, r \\
\frac{dr(t)}{dt} &= r - 2 \, M(r, v)
\end{aligned} \]

The space-time singularities are the **FIXED POINTS (FP)** of the geodesic equation:

\[ \begin{aligned}
\frac{dv_*(t)}{dt} &= 2 \, r_* = 0 \\
\frac{dr_*(t)}{dt} &= r_* - 2 \, M(r_*, v_*) = 0
\end{aligned} \]

The idea is to study the **linearized system** around the FPs.
Characterization of the singularity and integrability

Eigenvalues of the stability matrix $J$

$$\chi_{\pm} = \frac{1}{2} \left( \text{Tr} J \pm \sqrt{ (\text{Tr} J)^2 - 4 \det J } \right)$$

$$\text{Tr} J = 1 - 2 (\partial_r M)_{FP} \quad \text{det} J = 4 (\partial_v M)_{FP}$$

**Unstable spiral --- Black hole**

$$(\text{Tr} J)^2 - 4 \det J < 0$$

**Unstable node --- Naked Singularity**

$$(\text{Tr} J)^2 - 4 \det J \geq 0 \quad \det J > 0$$

**Singularity strength parameter**

$$S = \frac{(\partial_v M)_{FP} X_{FP}^2}{2}, \quad X_{FP} \equiv \lim_{(r, v) \to FP} \frac{v(r)}{r}$$

**The singularity is strong if $S > 0$, otherwise it is gravitationally weak**


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[Diagram showing the timeline of gravitational collapse and singularities from 1965-70 to 1984-86, with notes and equations related to the topic of cosmology and quantum gravity.]
Classical Kuroda-Papapetrou model

The origin $(0, 0)$ is a naked singularity if $\lambda \leq \frac{1}{16 G_0}$, and $S > 0$

RG-improved Kuroda-Papapetrou model

$$f_k(r, v) = 1 - \frac{2 G_0 m(v)}{r + \alpha \sqrt{\lambda}}, \quad \alpha = \frac{\xi^2 G_0}{\sqrt{4 \pi g_*}}$$

Corresponds to the generalized mass function

$$M_k(r, v) = G_k(r) m(v)$$

$$\det J \propto M(0, v) \propto G(0)$$

G(k) generates improper node singularities

$$\text{Tr} J = 1 - \frac{2 \lambda v_0 G_0}{\alpha \sqrt{\lambda}} \quad \det J \propto G(r) |_{r \to 0} = 0 \quad \text{Fixed Points line}$$

Strength: $S \propto G(0) = 0 \quad \text{Integrable}$

This result does not depend on the particular choice $k(r)$

Reminder

- Eigenvalues of the stability matrix
  $$\chi_{\pm} = \frac{1}{2} \left( \text{Tr} J \pm \sqrt{\left(\text{Tr} J\right)^2 - 4 \det J} \right)$$

- Strength of the singularity
  $$S = \frac{\dot{M}_{FP} X_{FP}^2}{2} \quad X_{FP} \equiv \lim_{(r, v) \to \text{FP}} \frac{v(r)}{r}$$
Summary

- We studied a RG-improved Kuroda-Papapetrou model;
- The only effect of a running Newton constant is to turn a strong naked singularity into a line of gravitationally weak singularities;
- The spacetime may be extended through the singularity;
- The integrability of the singularity does not depend on the regulator.